Nonlinear BEC Dynamics by Harmonic Modulation of s-wave Scattering Length*

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Overview

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  - Mean-field description
- Collective modes
  - Excitation of collective modes
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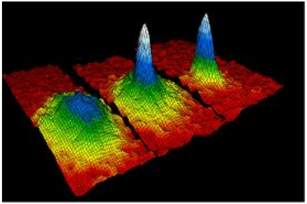
- Nonlinear features
  - Condensate dynamics
  - Excitation spectra
  - Analytic perturbative approach
- Conclusions

Experiments with ultracold atoms

- Nobel prize for physics in 2001 for the experimental achievement of BEC
- Cold alkali atoms:
  Rb, Na, Li, K . . .
  $T \sim 1\text{nK}$, $\rho \sim 10^{14}\text{cm}^{-3}$
- Cold bosons, cold fermions
- Harmonic trap, optical lattice
- Short-range interactions,
  long-range dipolar interactions



- Tunable quantum systems concerning dimensionality, type and strength of interactions

Mean-field description of a BEC

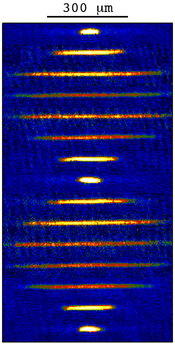
- BEC $\Rightarrow$ all atoms occupy the same state: $\psi(\vec{r},t)$ is a condensate wave-function
- Gross-Pitaevskii equation assuming $T=0$ (no thermal excitations)

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\Delta+V(\vec{r})+g|\psi(\vec{r},t))|^2\right]\psi(\vec{r},t)$$

- $V(\vec{r})=\frac{1}{2}m\omega_\rho^2(\rho^2+\lambda^2z^2)$ is a harmonic trap potential
- effective interaction between atoms is given by $g\times\delta(\vec{r})$
- $g=\frac{4\pi\hbar^2Na}{m}$, a is $\boldsymbol{s}$-wave scattering length, N is number of atoms in the condensate

BEC with modulated interaction

- Usually collective modes are excited by modulation of the external trap potential
- An alternative way of excitation - recent experiment by Hulet's and Bagnato's group: PRA **81**, 053627 (2010)
- BEC of ^{7}Li is confined in a cylindrical trap
- Time-dependent modulation of atomic interactions via a Feshbach resonance
- Interesting setup for studying nonlinear BEC dynamics



Gaussian approximation

- To simplify calculations and to obtain analytical insight, we approximate density of atoms by a Gaussian variational ansatz
- For a spherically symmetric trap

$$\psi^G(r,t) = \mathcal{N}(t) \exp\left[-\frac{1}{2}\frac{r^2}{u(t)^2} + ir^2\phi(t)\right]$$

- By extremizing corresponding action, we obtain an ordinary differential equation, PRL **77**, 5320 (1996)
- In the dimensionless form

$$\boxed{\ddot{u}(t) + u(t) - \frac{1}{u(t)^3} - \frac{p(t)}{u(t)^4} = 0}$$

- Interaction: $p(t) = \sqrt{\frac{2}{\pi}} Na(t)/l$, $l = \sqrt{\hbar/m\omega_\rho}$

Linear response

- Using this type of approximation and relying on the linear stability analysis, frequencies of low-lying collective modes have been analytically calculated
- The equilibrium width

$$u_0 = \frac{1}{u_0^3} + \frac{p}{u_0^4}$$

- Linear stability analysis

$$u(t) = u_0 + \delta u(t) \Rightarrow \delta\ddot{u} + \omega_0^2 \delta u = 0$$

$$\omega_0 = \sqrt{1 + \frac{3}{u_0^4} + \frac{4p}{u_0^5}}$$

Beyond linear response - motivation

- Due to the nonlinear form of the underlying GP equation, we have nonlinearity induced shifts in the frequencies of low-lying modes (beyond linear response)
- Our aim is to describe collective modes induced by harmonic modulation of interaction

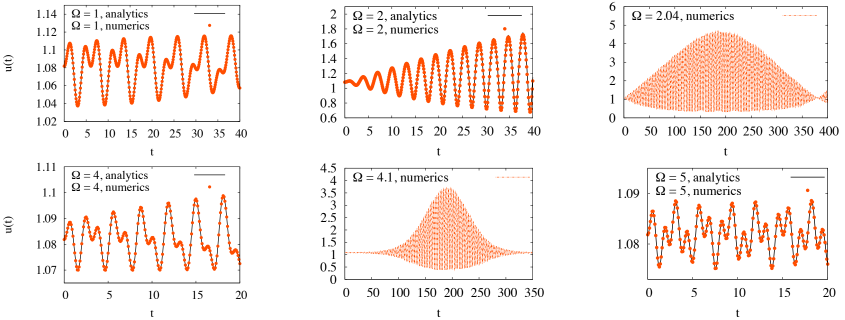
$$p(t) \simeq p + q\cos\Omega t$$

- q - modulation amplitude, Ω - modulation frequency
- For Ω close to some BEC eigenmode we expect resonances - large amplitude oscillations and role of nonlinear terms becomes crucial

Condensate dynamics

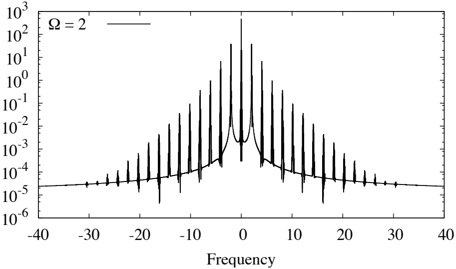
$$\ddot{u}(t) + u(t) - \frac{1}{u(t)^3} - \frac{p}{u(t)^4} - \frac{q}{u(t)^4}\cos\Omega t = 0$$

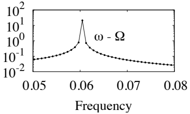
- $p = 0.4,\ q = 0.1,\ u(0) = u_0,\ \dot{u}(0) = 0,\ \omega_0 = 2.06638$
- Dynamics depends on Ω

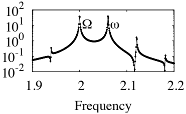


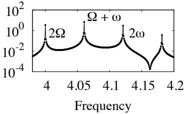
Excitation spectra (1)

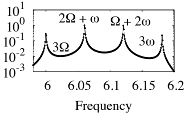
- We look at the Fourier transform of $u(t)$, $p = 0.4$, $q = 0.1$ and $\Omega = 2$





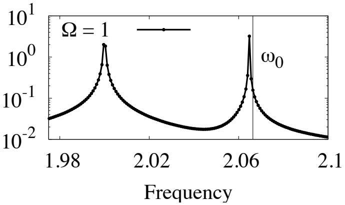


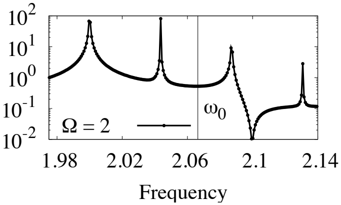




Excitation spectra (2)

- Frequency of the breathing mode is significantly shifted in the resonant region





Analytic perturbative approach (1)

- Linear stability analysis yields zeroth order collective mode ω_0 of oscillations around the time-independent solution u_0:

$$u_0 - \frac{1}{u_0^3} - \frac{p}{u_0^4} = 0, \qquad \omega_0 = \sqrt{1 + \frac{3}{u_0^4} + \frac{4p}{u_0^5}}$$

- To calculate the collective mode to higher orders, we rescale time as $s = \omega t$:

$$\omega^2 \ddot{u}(s) + u(s) - \frac{1}{u(s)^3} - \frac{p}{u(s)^4} - \frac{q}{u(s)^4} \cos \frac{\Omega s}{\omega} = 0$$

- We assume the following perturbative expansions in q:

$$\begin{aligned} u(s) &= u_0 + q\, u_1(s) + q^2\, u_2(s) + q^3\, u_3(s) + \ldots \\ \omega &= \omega_0 + q\, \omega_1 + q^2\, \omega_2 + q^3\, \omega_3 + \ldots \end{aligned}$$

Analytic perturbative approach (2)

- This leads to a hierarchical system of equations:

$$\begin{aligned}
\omega_0^2\,\ddot{u}_1(s) + \omega_0^2\,u_1(s) &= \frac{1}{u_0^4}\cos\frac{\Omega s}{\omega} \\
\omega_0^2\,\ddot{u}_2(s) + \omega_0^2\,u_2(s) &= -2\omega_0\,\omega_1\,\ddot{u}_1(s) - \frac{4}{u_0^5}\,u_1(s)\,\cos\frac{\Omega s}{\omega} + \alpha\,u_1(s)^2 \\
\omega_0^2\,\ddot{u}_3(s) + \omega_0^2\,u_3(s) &= -2\omega_0\,\omega_2\,\ddot{u}_1(s) - 2\beta\,u_1(s)^3 + 2\alpha\,u_1(s)u_2(s) - \omega_1^2\,\ddot{u}_1(s) \\
&\quad + \frac{10}{u_0^6}\,u_1(s)^2\,\cos\frac{\Omega s}{\omega} - \frac{4}{u_0^5}\,u_2(s)\,\cos\frac{\Omega s}{\omega} - 2\omega_0\,\omega_1\,\ddot{u}_2(s)
\end{aligned}$$

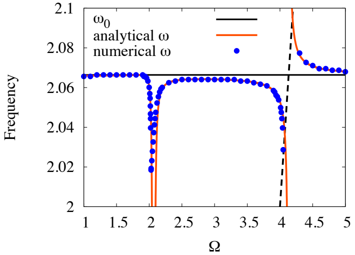
where $\alpha = 10p/u_0^6 + 6/u_0^5$ and $\beta = 10p/u_0^7 + 5/u_0^6$.

- We determine ω_1 and ω_2 by imposing cancellation of secular terms - Poincaré-Lindstedt method

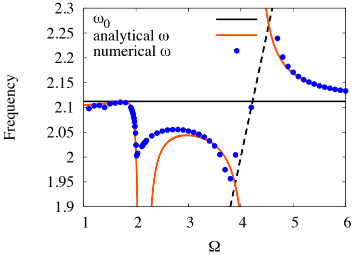
Results

- Frequency of the breathing mode vs. driving frequency Ω
- Result in the second order of the perturbation theory

$$\omega = \omega_0 + q^2 \frac{\text{Polynomial}(\Omega)}{(\Omega^2 - \omega_0^2)^2\,(\Omega^2 - 4\omega_0^2)} + \ldots$$



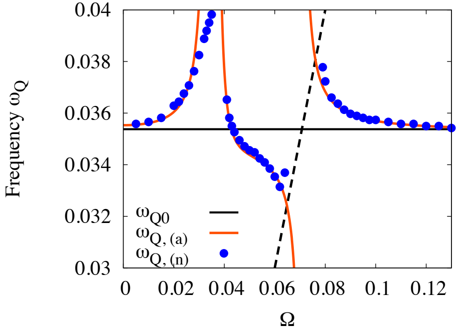
$p = 0.4, \quad q = 0.1$



$p = 1, \quad q = 0.8$

Experimental setup - results

- $p = 15$, $q = 10$, $\lambda = 0.021$,
  $\omega_{Q0} = 2\pi \times 8.2\,\mathrm{Hz}$, $\omega_{B0} = 2\pi \times 462\,\mathrm{Hz}$
- $\omega_B >> \omega_Q$, $\Omega \in (0, 3\omega_Q)$, large modulation amplitude
- Strong excitation of quadrupole mode
- Excitation of breathing mode in the radial direction
- Frequency shifts of quadrupole mode of about 10% are present



Conclusions

- Motivated by recent experimental results, we have studied nonlinear BEC dynamics induced by harmonically modulated interaction
- We have used a combination of an analytic perturbative approach, numerics based on Gaussian approximation and numerics based on full time-dependent GP equation
- Relevant excitation spectra have been presented and prominent nonlinear features have been found: mode coupling, higher harmonics generation and significant shifts in the frequencies of collective modes
- Our results are relevant for future experimental designs that will include mixtures of cold gases and their dynamical response to harmonically modulated interactions

Analytic perturbative approach (3)

- Secular term - explanation

$$\ddot{x}(t) + \omega^2 x(t) + C\cos(\omega t) = 0$$

$$x(t) = A\cos(\omega t) + B\sin(\omega t) - \underbrace{\frac{C}{2\omega} t\sin(\omega t)}_{\text{linear in } t}$$

- In order to have properly behaved perturbative expansion, we impose cancellation of secular terms by appropriately adjusting ω_1 and ω_2
- Another way of reasoning

$$u(t) = A\cos\omega t + A_1 t\sin\omega t \approx A\cos\omega t\cos\Delta\omega t + \frac{A_1}{\Delta\omega}\sin\Delta\omega t\sin\omega t$$

$$u(t) \approx A\cos[(\omega - \Delta\omega)t] \Rightarrow \Delta\omega = A_1/A$$