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Regime switching in coupled nonlinear systems: Sources, prediction, and control—Minireview and perspective on the Focus Issue

Special Collection: Regime switching in coupled nonlinear systems: sources, prediction, and control

Igor Franović ; Sebastian Eydam ; Deniz Eroglu

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Igor Franović,[1,a)] Sebastian Eydam,[2,b)] and Deniz Eroglu[3,4,c)]

AFFILIATIONS

[1] Scientific Computing Laboratory, Center for the Study of Complex Systems, Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia
[2] Neural Circuits and Computations Unit, RIKEN Center for Brain Science, 2-1 Hirosawa, 351-0198 Wako-shi, Japan
[3] Faculty of Engineering and Natural Sciences, Kadir Has University, 34083 Istanbul, Turkey
[4] Department of Mathematics, Imperial College London, London SW7 2AZ, United Kingdom

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[a)] **Author to whom correspondence should be addressed:** franovic@ipb.ac.rs
[b)] **Electronic mail:** richard.eydam@riken.jp
[c)] **Electronic mail:** deniz.eroglu@khas.edu.tr

ABSTRACT

Regime switching, the process where complex systems undergo transitions between qualitatively different dynamical states due to changes in their conditions, is a widespread phenomenon, from climate and ocean circulation, to ecosystems, power grids, and the brain. Capturing the mechanisms that give rise to isolated or sequential switching dynamics, as well as developing generic and robust methods for forecasting, detecting, and controlling them is essential for maintaining optimal performance and preventing dysfunctions or even collapses in complex systems. This Focus Issue provides new insights into regime switching, covering the recent advances in theoretical analysis harnessing the reduction approaches, as well as data-driven detection methods and non-feedback control strategies. Some of the key challenges addressed include the development of reduction techniques for coupled stochastic and adaptive systems, the influence of multiple timescale dynamics on chaotic structures and cyclic patterns in forced systems, and the role of chaotic saddles and heteroclinic cycles in pattern switching in coupled oscillators. The contributions further highlight deep learning applications for predicting power grid failures, the use of blinking networks to enhance synchronization, creating adaptive strategies to control epidemic spreading, and non-feedback control strategies to suppress epileptic seizures. These developments are intended to catalyze further dialog between the different branches of complexity.

Maintaining regular function, flexibility, and adaptive capability in complex systems, as well as preventing dysfunction or collapse, often depends on regime switching triggered by changes in the systems' conditions. In this Focus Issue, regime switching is introduced as a broad paradigm referring to qualitative shifts between long-term dynamical states, which may not be asymptotically stable. These shifts can be sudden or gradual, reversible or irreversible, and can occur as single events or be parts of sequential activity patterns. In this way, the concept encompasses phenomena such as tipping, heteroclinic switching, noise-induced attractor hopping, chaotic itinerancy, and quasi-stationary states in adaptive networks. Key theoretical challenges include reducing the effective dimensionality of collective dynamics, resolving the impact of multiple timescales, and extending bifurcation theory to non-autonomous systems. This Focus Issue seeks to advance reduction methods using frameworks like Ott–Antonsen (OA) and Montbrió–Pazó–Roxin (MPR), as well as to promote the integration of stochastic and multiple timescale approaches to time-varying networks. Another key aspect concerns improving our understanding of the role of multistability and chaotic saddle invariant sets in the organization of regime switching, especially in noisy environments. Given that regime switching can lead to catastrophic failures or extreme events, enhancing capabilities in their detection, prediction, and control is essential. This Focus

Issue underscores progress in reduction approaches, data-driven detection methods, and control strategies, supporting interdisciplinary exchange in fields like neuroscience, gene regulatory networks, population dynamics, laser dynamics, power grids, and extreme events.

I. INTRODUCTION

Over the past two decades, advances in big data assimilation and analysis have provided deep insights into how transitions between qualitatively distinct dynamical states give rise to optimal operation, flexibility, adaptation, or even dysfunction and collapses in complex systems. While some transitions have attracted much attention for their severe and long-lasting impact, like the ice sheet degradation,[1] loss of Arctic sea ice,[2] breakdown of the Atlantic Meridional Overturning Circulation,[3] Amazon rainforest dieback,[4] or the disruption of food chains due to species extinction,[5] other switching phenomena seamlessly blend into our everyday lives, shaping motor coordination,[6] working memory,[7] decision making,[8] or gene expression.[9] Explaining such processes has prompted renewed interest in generalizing to more complex scenarios of the classical bifurcation theory, which traditionally addresses qualitative changes in low-dimensional autonomous or periodically forced systems under adiabatic parameter variation. In this context, several key issues have emerged, including the reduction of effective dimensionality of systems' dynamics to allow for the use of bifurcation theory, advances to the theory of (stochastic) multiple timescale processes, and the extension of attractor and other concepts from bifurcation theory to non-autonomous systems. Additionally, there has been a growing need for new methods to study temporal dynamical networks under different ratios of characteristic timescales involved and for the improved understanding of the role of saddle invariant sets and metastable states in organizing transient and long-term dynamics of complex systems. Finally, machine learning-based techniques are being developed to detect and predict transitions in real-world systems, while in parallel, new efficient control strategies are devised for systems where the application of feedback may not be feasible. The described challenges often intersect: for instance, systems with multiple timescale dynamics can exhibit high-dimensional slow manifolds, or the noise may act in concert with a fast varying parameter to significantly alter the systems' behavior. Overcoming such challenges is relevant not only to enhancing our theoretical understanding but also for developing practical solutions to pressing real-world problems.

To cover the wide scope of the stated problems, we introduce the term *regime switching* as a general paradigm to describe transitions occurring in complex systems between qualitatively distinct long-term dynamical states, referred to as regimes. Certain aspects of this definition are deliberately not specified or are subtly differentiated from the related phenomena, such as tipping, also called critical transitions or regime shifts depending on the particular branch of complexity. Notably, the regimes per se are not required to exhibit asymptotic stability, but should just be sufficiently long-lived to be observable. Also, no assumptions are made on the timescale of transitions compared to the characteristic timescale of local dynamics (relaxation or dissipation rate), allowing not only for abrupt but also for more gradual transitions found, e.g., in pattern-forming[10] and heterogeneous systems,[11,12] or the process of species extinction.[13] Unlike classical tipping, regime switching may be triggered by parameter changes or forcing that are not necessarily small or cumulative. Moreover, apart from occurring as isolated events, these transitions can also be parts of sequences, such as cascades or cyclic patterns found in climate models,[14,15] coupled oscillators,[16–18] or neuroscience.[19–23] As a process, regime switching can be reversible, including smooth (continuous) or explosive (sudden and discontinuous) transitions, or may be irreversible, marked by jump-like hysteretic shifts to contrasting regimes.[24] Conceptually, the regime switching framework described above seeks to unify a broad range of phenomena, including (i) all forms of *tipping*, (ii) *heteroclinic switching* in single or coupled heteroclinic cycles (heteroclinic networks), (iii) *noise-induced switching* (*attractor hopping*) between the coexisting metastable states, (iv) *chaotic itinerancy* between quasi-attractors, and (v) *sequential activity patterns* in adaptive networks. Though not all of these subclasses are discussed in depth in this Focus Issue, each is briefly considered in Sec. II.

A significant area of research related to regime switching is the development of reduction approaches, which simplify the systems' high-dimensional dynamics into effective low-dimensional models. These methods have been instrumental in various fields, ranging from mean-field models in condensed matter physics,[25] over master stability function (MSF) in coupled oscillators[26] to neural mass models in neuroscience.[27] The cornerstone for the current expansion of reduction techniques has been laid by the Ott–Antonsen (OA)[28,29] and Watanabe–Strogatz (WS)[30,31] frameworks. They have provided a mathematically rigorous theory resolving among else, a long-standing issue of why many oscillator population models display low-dimensional collective behavior. More recently, next-generation neural mass (NGNM) models, prompted by the work of Montbrió *et al.*,[32] have provided new insights on states of partial synchrony and multistability, adapting the OA reasoning to populations of quadratic integrate-and-fire neurons. As their key advantage, all three recent reduction approaches hold exactly, allowing for the study of regime switching by classical bifurcation theory. This Focus Issue contributes to the field in several directions, including the understanding of the impact of non-Gaussian white noise,[33,34] finite-size effects,[35,36] and adaptation.[37] Another branch of reduction approaches that has witnessed recent progress concerns multiple timescale dynamics,[38] in particular, in relation to the impacts of forcing, adaptation, and noise. This trend is also reflected in this Focus Issue by explaining the onset and multistability of multimode bursting in a motif of Bonhoeffer–van der Pol oscillators with a slow periodic drive.[39] We further hope to encourage the combination of reduction approaches, such as NGNM or OA methods with slow–fast reduction,[22,40,41] which may prove valuable across a wide range of complex systems, including, e.g., for adaptive and other temporal dynamical networks.

While reduction approaches focus on simplifying high-dimensional dynamics into effective low-dimensional models, another crucial aspect of understanding regime switching relates to data-driven methodologies. The latter provide practical tools for data compression to extract meaningful statistical and other hidden structures of systems' dynamics from complex, high-dimensional datasets.[42,43] In many applications, model-free detection and forecasting of regime switching from time series is often complicated by high dimensionality, irregular sampling times, and noise from various sources.[43] Overcoming these difficulties is highly important because regime switching can sometimes lead to undesired states harmful to the system function or can even trigger systemic failures. For instance, tipping phenomena may be accompanied by extreme events or transitions to regimes where extreme events become more frequent.[43] Therefore, identifying precursors, i.e., early-warning indicators of critical transitions, has become essential for timely interventions that could avert or abate dangerous outcomes or allow enough time to devise mitigation strategies. This Focus Issue features several contributions on detecting regime switching using data-driven approaches, including topological[44] and recurrence analysis,[45] causation entropy boosting,[46] structural changes in functional neural network,[47] and machine learning techniques.[48,49] Notably, despite the recent advances in early-warning indicators of tipping via deep learning[50–53] and reservoir computing,[54] contributions in these directions were not submitted to this Focus Issue.

Significant progress has been made not only in understanding regime switching through theoretical frameworks and data-driven methodologies but also by developing effective control strategies to induce the desired or suppress the potentially harmful transitions. The latter is essential to achieving and maintaining optimal function in complex systems and often hinges on modifying the systems' multistable behavior.[55] While multistability can provide operational flexibility, it may also interfere with optimal performance. Control strategies can have different objectives, including stabilizing certain states against noise-induced switching, guiding transitions toward target states, and altering the state space landscape to eliminate the undesired states or to affect the preference of attractors. In some real-world systems, including the brain, cellular automata or reservoir computers, maintaining function in the vicinity of a transition, e.g., at the "edge of criticality"[56,57] or "edge of chaos,"[58,59] or managing long chaotic transients are also critical goals. Control theory classically distinguishes between feedback, non-feedback, and stochastic control methods.[55] Nevertheless, the difficulties in applying feedback in many complex systems[60] has gradually shifted the focus toward non-feedback and stochastic approaches, as well as the emerging field of control via adaptive[61] and other time-varying networks.[62] Promising strategies include less invasive non-feedback techniques, such as parameter perturbations, and stochastic control methods aimed at selecting or eliminating specific regimes. Given the evolving landscape of complex systems, greater emphasis is needed on controlling sequential switching phenomena, such as heteroclinic cycles. This Focus Issue features key contributions addressing control dynamics, including (i) chaos control—examining transitions between regular and chaotic behaviors,[63] including spatially localized chaos;[64] (ii) time-varying networks for managing transitions to and from synchronization[65,66] and the onset of recurring extreme events;[67] (iii) control by adaptation for epidemic spreading[68] and population dynamics;[69] (iv) stochastic control in metapopulation models[70] and power grids;[48,71] and (v) non-feedback control strategies for epileptic seizure dynamics through external pulses and perturbations of parameters and dynamical variables.[37]

In summary, regime switching represents a critical area of research in understanding complex systems. Theoretical frameworks, such as reduction approaches, offer valuable insights into the underlying dynamics, while data-driven methodologies enhance detection and prediction capabilities. Furthermore, control strategies are essential for managing regime transitions, thereby promoting the desired states and suppressing the undesirable ones. This Focus Issue addresses these key aspects, highlighting the recent advances and aiming to catalyze further developments in the field.

II. MULTIPLE FACETS OF REGIME SWITCHING

Regime switching in complex systems manifests in various forms, each presenting unique challenges related to their different dynamical background. This section surveys some of the most prominent mechanisms driving these transitions, highlighting their theoretical foundations and practical implications. Broadly speaking, regime switching may comprise isolated transitions between asymptotically stable regimes, or the transitions may foster sequential or cyclic activity patterns involving a succession of long transients. In the following, we summarize both types of phenomena, singling out tipping as a representative of the former, and discuss several more subtle paradigms related to the latter class, including heteroclinic switching, chaotic itinerancy, attractor hopping, and sequential switching triggered by adaptation. These multiple facets underline the complexity of regime switching and emphasize the range of methods required to understand, detect, predict, and control such transitions.

A. Tipping

In the last two decades, a plethora of different tipping phenomena has been described in different branches of complexity science, from Earth's climate (ice sheet loss, permafrost melting, disruption of monsoons, collapse of Atlantic Meridional Overturning Circulation), ecology (Amazon forest dieback, desertification, and extinction of species), and medicine (asthma and migraine attacks, onset and termination of epileptic seizures, cardiac arrest), to the social sphere (market crashes, power blackouts, mass panic, forming of public opinion).[72,73] Tipping has classically been described as an abrupt and persistent shift to a qualitatively different dynamical state by small and gradual changes in the system's external conditions, including parameter variation, forcing, and noise, which may act independently or in concert.[74] Though the terms critical transitions, tipping and regime shifts are nowadays used interchangeably, historically their scopes had subtle differences due to their distinct background. While the term tipping has maintained the highest level of generality, though at first often implying irreversibility of the regime change, the term critical transitions has originally been reserved for tipping under variation of an external parameter, drawing analogy to phase transitions. The emphasis on the small amplitude of perturbations, which trigger the transition derives from the initial understanding of the regime shifts in ecology, implying an inherent self-amplifying character of the tipping mechanism via positive feedback.[75]

At the earlier stage, there have been many attempts to classify tipping phenomena by drawing analogies to bifurcation theory on one hand, and to the theory of phase transitions, on the other hand. However, not all instances of tipping are related to bifurcations, and also, not all of them involve ergodicity and symmetry breaking inherent to phase transitions. Still, tipping phenomena manifest certain universal features, like being associated with exceeding certain thresholds, such as boundaries between (fractal) attraction basins in multistable systems, bifurcation points, or critical rate of parameter variation. In these terms, Ashwin *et al.*[76] classified tipping phenomena to three categories, namely, B-induced tipping associated with a slow passage through a bifurcation threshold; N-induced tipping, conforming to a noise-induced escape from the vicinity of a metastable state (though some definitions invoke stochastic bifurcation[77] or refer to a switch to a coexisting metastable state[78]); and R-induced tipping in non-autonomous systems, where the dynamics fails to track the attractor adiabatically due to a too fast parameter variation. A later addition to this classification concerns shock-induced tipping triggered by the large perturbations of external parameters or the systems' variables.[73] While B- and N-tipping can be qualitatively explained by the concepts from classical bifurcation theory, most notably that of multistability,[72,73] R- and shock-induced tipping[79] require a more general framework. In particular, the study of R-induced tipping gave birth to the notion of local-pullback (snapshot) attractors as the generalization of the attractor concept to non-autonomous systems.[80,81] More recently, the requirement for the transition abruptness relative to the characteristic timescale of the states' dynamics has been relaxed in light of gradual tipping observed in pattern-forming systems,[10,12] which unfolds by a succession of intermediate regimes or by front propagation separating the different coexisting patterns, then the scenarios with a high topological complexity,[82] and the systems with modular structure undergoing tipping cascades.[14,83] Concerning the dependence on multiple timescale dynamics, a particular relation between the system's relaxation time and the timescale of parameter variation may make the tipping onset fast or slower,[84,85] and for the latter, a too short transgression over the threshold may not even trigger tipping.[86] Regarding the nature of the associated states, recent extensions concern cases where the attractors are more complex than equilibria (periodic or chaotic attractors),[82] whereby the tipping often becomes phase-sensitive.[87] Another scenario may involve states that are not attractors, such as R-tipping in excitable systems.[88]

Tipping phenomena may give rise to systemic failures and may be accompanied by extreme events or involve transitions to regimes with their more likely recurrence.[43] Thus, it has become a necessity to develop early-warning indicators of tipping which could allow for timely application of potential control strategies to avert or soften the pending dangerous and catastrophic events, or to provide enough time to devise mitigation strategies. The initial success of theory-driven approaches in deriving generic early-warning indicators for bifurcation-related tipping has been prompted by the analysis of the reduced, "normal form"-like low-dimensional (stochastic) multiple timescale models,[77,89] portraying a tipping system with a near zero real part of the dominant eigenvalue as an overdamped particle in an ever softening potential well.[90] The reduced ability to recover from local perturbations manifests as critical slowing down,[91–94] with the accumulation of perturbations reflected in the increase of lag-1 autocorrelation, variance or the asymmetry of fluctuations (skewness).[95] Nevertheless, the robustness and reliability of these indicators for empirical time-series, and even model systems, have been limited by the high dimensionality of data, dependence on the type of bifurcation, and the sensitivity to noise beyond white and Gaussian.[96–98] Other methods for detecting and forecasting tipping have invoked concepts from non-equilibrium phase transitions (discontinuity of non-equilibrium entropy or heat capacity),[99] and network theory (visibility graphs, recurrence methods, transition networks)[42] or have aimed for evaluation of the dominant eigenvalue based on Takens's embedding theorem.[100] Nevertheless, the most successful methods so far leverage machine learning tools, most notably deep learning.[50,52,53] The latter have demonstrated the ability to determine the type of bifurcation and to predict the critical point, though with potential caveats related to the dependence on training data sets.[100] These tools offer a more flexible approach to forecasting tipping, with the potential for broader application across various fields.

B. Heteroclinic switching

In many fields of complexity, from the brain to ecosystems, activity is organized in sequential activity patterns that are generic, i.e., reproducible and robust, and yet consist of just long transients connected by rapid switching events. Their dynamical background may be different, and understanding their self-organization requires nonlocal stability analysis. One of the most influential concepts for sequential activity is based on heteroclinic cycles,[101] which consist of saddle invariant sets connected by heteroclinic orbits. Apart from asymptotic stability, when their period increases with time and tends to infinity,[102] they may also display non-asymptotic stability.[103] Heteroclinic cycles generically organize periodic or chaotic global dynamics in systems with symmetry,[104] such that saddles correspond to patterns of localized activity or partial synchrony. Earlier models have typically considered phase-locking patterns in frequency-synchronized phase oscillators where saddle equilibria correspond to phase-locked cluster states.[16] More recently, the focus has shifted to switching between saddle chaotic sets, which conform to patterns of localized frequency synchrony (weak chimeras) in coupled populations of identical oscillators with higher-order interactions.[17,104–106]

Apart from model systems, heteroclinic cycles have often been considered in the context of neuroscience. Earlier work has related them with the activity of network motifs featuring winnerless competition,[19,107] such as central pattern generators. Nevertheless, encoding along sequences of saddles has also been invoked as a paradigm for more complex scenarios, such as information presentation in visual, auditory, and odor perception,[19,108] as well as the organization of spontaneous sequential activity of neuronal populations in rest-state networks.[109] The concept has recently been brought back to the spotlight by suspected links to higher cognitive functions, such as creativity and its interplay with emotions.[20,110,111] Moreover, coupled heteroclinic cycles are gaining attention (heteroclinic networks),[102,103] which naturally emerge in neuroscience for coupled network motifs or modular and hierarchical neural networks.[23,112]

C. Chaotic itinerancy

Chaotic itinerancy[21,113] is a form of closed-loop activity between the long-term transients, where the switching between the long epochs of low-dimensional and nearly periodic dynamics occurs via short high-dimensional chaotic transients. The pseudo-stationary dynamics unfolds within the so-called attractor ruins nearby the quasi-attractors,[114] which are envisioned as dynamical objects with a neutral stability, such that they attract a positive measure of orbits but still possess no asymptotic stability. In contrast to metastable states, an escape from their vicinity does not require a finite but only an infinitesimal perturbation. Chaotic itinerancy has classically been invoked in relation to neuroscience,[21,115] most often as a paradigm for associative memory or for the spontaneous cortical activity underlying fast switches from synchronized to desynchronized behavior but has also been associated with pattern formation in model systems.[116]

D. Attractor hopping

Attractor hopping[117] is a form of noise-induced sequential activity that comprises the noise-triggered switching between the coexisting metastable states derived from the attractors of the noiseless system. Hopping in systems with high multistability and fractal basin boundaries is qualitatively different from that in bistable ones, because the structure of the saddles may determine which transitions are allowed.[117] Moreover, in highly multistable systems, sufficiently strong noise may bias the switching process by affecting the attractor preference. This effect, called noise-induced preference of attractors,[55] manifests as the washing out of metastable states associated with attractors whose basins are small and are thereby most fragile to noise. In terms of applications, slow switching between metastable states triggered by weak fluctuations is believed to play an important role in various fields, including chemical kinetics,[118] gene regulatory networks,[9,119] population dynamics,[120] neural networks,[121,122] and transitions between the different states of partial synchrony.[123]

E. Sequential activity in adaptive networks

Adaptation has quite recently emerged as a source of robust sequential activity involving rapid transitions between the quasi-stationary states of partial synchrony. Though their self-organization mechanisms are different, these patterns still have two common features: (i) they are facilitated by adaptive interactions; (ii) they are slow–fast phenomena and require adaptive couplings to evolve on a timescale slower than that of local dynamics. So far, four different models of adaptive networks have been reported to generate sequential activity, including *recurrent synchronization*[22] in networks with asymmetric adaptation rules, which consists of periodic alternation between the synchronous (phase-locked) and asynchronous (typically frequency clustered) quasi-stationary states; *synchronization cluster bursting*, where different states of partial synchronization form and dissolve in a burst-like fashion;[124] *chaotic recurrent clustering*,[18] where the switching between the different frequency clusters is irregular involving slow chaos in coupling dynamics; *canard cascading* in networks with global adaptive coupling,[125] organized by a robust heteroclinic canard cycle where sequences of quasi-stationary states conform to slow motion along the saddle slow manifolds. Featuring different types of local dynamics (phase oscillators,[18,124] neural oscillators,[22] and semiconductor lasers[125]), these models suggest a promising avenue for various applications, especially to neuromorphic computing.

III. REDUCTION APPROACHES

Reduction approaches have attracted considerable attention within the last decade, playing an important part in the recently gained theoretical insights into the collective dynamics of high-dimensional complex systems. The advent of methods that under certain conditions hold exactly rather than approximately has enabled low-dimensional reductions that accurately account for the collective dynamics, allowing for the use of classical bifurcation theory to study the stability of complex systems and their sensitivity to perturbations. The reduction approaches address various aspects of complexity. In particular, Ott–Antonsen,[28,29] Watanabe–Strogatz,[30,31] and Montbrió–Pazó–Roxin[32] approaches treat the states of partial synchronization in populations of coupled oscillators and neurons; master stability function[26] concerns the local stability of the completely synchronized state; the dimension reduction[126,127] approximates the dynamics of complex systems near tipping, and the multiple timescale reductions[128] allow for the separation of the dynamics unfolding on the fast or slow characteristic timescales. In the following, we make a brief overview of these methods, highlighting the important contributions of this Focus Issue.

A. Ott–Antonsen and Watanabe–Strogatz reductions

The explanation for frequent observations of robust low-dimensional collective dynamics in populations of coupled oscillators, despite their high-dimensional phase spaces, has long remained elusive. A rigorous mathematical understanding arrived with the Watanabe–Strogatz (WS)[30,31] and Ott–Antonsen (OA)[28,29] theories, which in parallel triggered a shift of paradigm from the synchronization transition to the study of complex states of partial synchrony. The essential finding of the WS approach is that the dynamics of finite assemblies of identical phase oscillators sine coupled to a common forcing, which can be the mean field, exactly reduces to the dynamics of three macroscopic (WS) variables plus constants of motion. On the other hand, the OA theory holds exactly in the thermodynamic limit for heterogeneous populations of oscillators with a Cauchy distribution of natural frequencies. Its main result concerns the existence of an invariant two-dimensional (OA) manifold that corresponds to a wrapped Cauchy distribution of phases, such that systems' dynamics generically converges to it for asymptotically long times.[129] On this manifold, the dynamics reduces to just a single complex equation for the global order parameter. The eventually established relation between the WS and OA approaches has indicated that the OA reduction conforms to the particular choice, namely, the uniform distribution, of the constants of motion for the WS reduction.[130,131] More recently, it has been shown that both approaches can be generalized to populations influenced by local Cauchy noise.[132,133] Nevertheless, the scenarios with more general forms of noise remain a field of intense study.

In the latter context, this Focus Issue features an extension of the OA formalism to systems featuring non-Gaussian white noise. In particular, Dolmatova *et al.* employ a circular cumulant approach to generalize OA theory to coupled phase oscillators with local α-stable white noise,[34] which is frequently encountered in financial markets, biological, and physical systems. The validity of the derived two-cumulant reduction is illustrated for the Kuramoto synchronization transition, as well as Abrams scenario[134] giving rise to chimeras in hierarchical populations of stochastic phase oscillators.

B. Next-generation neural mass models

The recent arrival of the next-generation neural mass (NGNM) models, spurred by the seminal work of Montbrió *et al.*,[32] has allowed for a systematic analysis of multistability and switching dynamics in neuronal populations (see Refs. 131 and 135–137 for comprehensive review). Earlier phenomenological neural mass and neural field models[138,139] approximated the population firing rate dynamics by developing heuristic arguments for the static, typically sigmoid-shaped transfer function between the mean membrane potential and mean firing rate, invoking assumptions such as negligible correlations in local dynamics. In sharp contrast, the NGNM models characterize population dynamics in terms of mean-field equations for the mean firing rate and the mean membrane potential which are exact in the thermodynamic limit. The NGNM framework is based on the Lorentzian Ansatz for the populations of quadratic integrate-and-fire neurons, describing the evolution on the OA manifold by variables adapted for neuroscience. Note that an alternative approach employing explicitly the OA Ansatz[28,29] has been developed for populations of theta neurons.[140] The NGNM models can be used to trace the transitions between the different states of partial synchrony via standard bifurcation analysis, also providing insights into the microscopic structure of the associated collective states. While the classical notion of spike synchrony between neurons is reflected in the onset of collective oscillations,[141] the NGNM framework further establishes a conformal mapping between the mean-field variables and the level of population synchrony described by the complex Kuramoto order parameter.[32,136] So far, NGNM models have been used to address a broad spectrum of theoretical problems, from the impact of different types of synaptic coupling,[142–145] synaptic noise,[129,146] quenched randomness,[146] and finite-size effects,[35,147] over the onset of collective rhythms in (coupled) neuronal populations[141,148–151] and spatial patterns in neural fields,[152–154] to experiment-related studies of cognitive[155,156] and pathological processes[157,158] and simulations of the whole brain activity.[159,160]

This Focus Issue provides advances to the NGNM framework in several directions, including the impact of non-Gaussian additive noise, the finite-size effects, and metabolic feedback to the dynamics of populations of quadratic integrate-and-fire neurons.

In particular, Goldobin *et al.* develop a new reduction theory based on pseudocumulant expansion for the population of quadratic integrate-and-fire neurons influenced by local α-stable white noise.[33] This type of non-Gaussian noise, characterized by heavy-tail power-law distributions, is argued to emerge generically in diverse scenarios, including, e.g., large populations with imperfect synchrony, the impact of finite-size effects, or sparse connectivity. Remarkably, the analysis for the case of fractional α has revealed that the minimal asymptotically rigorous reduction for the case of additive Gaussian noise $\alpha = 2$ has to include three rather than two pseudocumulants, which has been the practice so far. Apart from implications to neuroscience, the new formalism can also apply to condensed matter physics, like the problem of Anderson localization in one-dimensional systems.

Klinshov *et al.* consider a form of additive common noise, called shot noise, to capture the impact of finite-size effects on the bifurcation structure and switching dynamics in a population of excitatory quadratic integrate-and-fire neurons.[35] Fluctuations derived from the system's finite size turn the attractors of the related NGNM model into metastable states and are capable of triggering switching dynamics, manifested as slow stochastic fluctuations between the coexisting metastable states. Shot noise is further demonstrated to shift the bifurcation structure compared to the thermodynamic limit, playing, in some cases, a constructive role by expanding the parameter domains that support certain regimes.

Kirillov *et al.* consider the thermodynamic limit dynamics and finite-size effects for a binary cortical motif of an excitatory and inhibitory population of quadratic integrate-and-fire neurons with bidirectional inter-population coupling.[36] Despite its simplicity, the model supports different types of multistable behavior, including the coexistence of high- and low-activity macroscopic stationary states and an oscillatory mode, as well as bistability between two periodic or two chaotic regimes. While periodic solutions are typically born from the Hopf instability of the high-activity stationary state, chaos is shown to emerge via the Feigenbaum scenario. The finite-size effects, captured by shot noise, induce fluctuations around the thermodynamic limit attractors, as well as the recurrent switching between the coexisting metastable states. Interestingly, certain subtle structures from the thermodynamic limit, such as periodic windows within chaotic domains, remain relatively well preserved in large but finite populations.

Eydam *et al.* build an NGNM model to capture the impact of metabolic feedback associated with a ketogenic diet, explaining the scenarios for transitions to synchrony and suggesting strategies to control the switching dynamics between the asynchronous and seizure-like synchronous states[37] (see also Sec. VI B). Contrasting classical NGNM models, their system is three-dimensional due to an additional equation for the dynamics of mean ATP concentration, which reflects the metabolic feedback. Depending on the coupling strength, two qualitatively different scenarios for the transition to synchrony are revealed. For weaker couplings, a bistability region between the lower- and the higher-activity asynchronous states emanates from the cusp point, and collective oscillations emerge via a supercritical Hopf bifurcation. For stronger couplings, a complex bifurcation scenario is revealed, involving seven co-dimension two bifurcation points, including pairs of Bogdanov–Takens and generalized Hopf points. The latter allows both the lower- and the higher-activity asynchronous states to undergo transitions to synchrony, with hysteresis observed in the vicinity of subcritical Hopf bifurcations.

C. Master stability function approach to network synchrony

Synchronization is one of the most important paradigms of self-organization in natural and synthetic complex systems, from biology, physiology, and neuroscience, over ecology, social behavior, and power grids to physics, chemistry, and technology.[161] The linear asymptotic stability of the synchronization manifold in networks of identical oscillators is classically analyzed by the method of master stability function (MSF), which facilitates a high-dimensionality reduction and provides a unified framework for studying complete synchronization independently of the oscillators' nature. MSF involves the separation of dynamical and topological features of the network, relating the stability of the fully synchronized state with the spectrum of the network's graph Laplacian. Since the discovery by Caroll and Pecora,[26] the MSF approach has been extended to various directions, accommodating on one hand for interaction delays[162,163] and more complex interaction patterns, e.g., multilayer[164] and multiplex networks,[165] while on the other hand, facilitating the analysis of local stability of cluster states.[166] More recently, the method has also been generalized to adaptive networks,[167] static and time-varying networks with higher-order interactions,[168–170] and simplicial complexes.[171,172]

This Focus Issue highlights the application of MSF to time-varying networks. In particular, in two papers, the MSF method is employed as a tool to optimize the form and structure of interactions in time-varying networks of chaotic oscillators to induce and improve the stability of the completely synchronized state (see also Secs. VI A and VI D). While Dayani *et al.* use the MSF framework to select the optimal time-varying coupling between the different local variables,[65] Sivanagesh *et al.* consider an alternative strategy based on temporary decoupling of oscillators.[66]

D. Dimension reduction for dynamical networks

Representation of many high-dimensional complex systems by networks sparked interest into developing effective low-dimensional models of network dynamics that can predict the critical point, their response to perturbations, and the onset of tipping-related correlations. Motivated by the dimension reduction of high-dimensional data, the immediate goal has become to determine whether certain projections of high-dimensional network dynamics, such as sums of weighted local variables, can approximate without substantial information loss the network dynamics near criticality. To do so, Gao *et al.* introduced a one-dimensional reduction,[126] demonstrating for the class of directed weighted networks with negligible degree correlations, that an appropriately weighted sum of local variables can approximate the resilience function of network systems. The latter characterizes their ability to maintain normal operation in the presence of perturbations, taking into account the nodal in- and out-degrees. Such degree-based mean-field theory implies that nodes with the same degree statistically display similar dynamics, such that the tipping dynamics is primarily influenced by the nodes with high connectivity degrees. More recently, the one-dimensional reduction has been extended to more general types of topology by the spectral method, leveraging the eigenvectors and eigenvalues of the networks' adjacency or related matrices.[127,173] While the near-tipping resilience function is best approximated using the dominant eigenvector of the adjacency matrix,[174] it has turned out that non-leading eigenvectors may help in optimizing the prediction error of networks' collective observables.[175] In parallel, a two-dimensional reduction based on the spectral method has been developed to accurately predict the tipping point.[176]

In this Focus Issue, Ghosh *et al.*[177] employ a resilience function and the reduction based on eigenvector centrality to investigate the contagious process on networks with triadic interactions within the susceptible-infected-susceptible class of models. Using the resilience function, insight is gained into the local structure of stationary states, in particular, the scaling between the stable healthy states of nodes with their degree. Also, group interactions are shown to impact the character of macroscopic transitions from the non-healthy state, giving rise to discontinuous transitions. The reduction based on eigenvector centrality has been used to calculate the critical point for the onset of epidemics. Both applied reduction techniques are indicated to have a potentially wide range of applications, from ecology to gene regulatory networks.

E. Reduction in multiple timescale systems

Systems with multiple timescale dynamics are abundant both in nature and technology, with examples ranging from intrinsic dynamics of neurons and cardiac cells, the dynamics of neuronal populations with synaptic plasticity, enzyme kinetics and reaction–diffusion systems, over social interactions and weather forecasting, to lasers or neuromorphic computing,[38,128] to name but a few. From the dynamical perspective, separation of timescales may be associated with non-autonomous systems or may derive from coupling delay or feedback, slow parameter drift, adaptation, learning, structural complexity of systems' self-organization, and other reasons. The most often invoked framework to study systems with multiple timescale dynamics is singular perturbation theory.[38,128] The typical scenario refers to so-called slow–fast systems of ordinary differential equations with two characteristic timescales where taking to zero the small parameter that accounts for scale separation translates the system to a different structural class. Other instances may include partial differential equations, stochastic differential equations, piecewise-smooth differential equations, or discrete-time systems.

Following the classical results of Tikhonov and Fenichel, it has long been known that the condition of normal hyperbolicity allows for the use of geometric singular perturbation theory,[128] which facilitates the reduction of slow–fast systems to singular limit problems on the fast and slow timescales, called layer and reduced problem, respectively. Within the layer problem, the goal is to determine the attractors of the fast flow treating slow variables as parameters. The stable equilibria obtained for infinite scale separation then comprise the critical manifold, which approximates well the slow invariant manifold for large but finite scale separation. The reduced problem consists in deriving the dynamics of the reduced (slow) flow either by applying adiabatic elimination for the stable equilibria of the fast flow, or by employing the averaging approach for the stable fast oscillations.

However, the normal hyperbolicity condition is often violated due to non-hyperbolic points or singularities on the critical manifold. This leads to more intricate phenomena, such as canards, which are critical for the onset of relaxation or mixed-mode oscillations.[178] While the loss of normal hyperbolicity in deterministic systems has been mitigated by various techniques, such as geometric desingularization (blowup method),[179] the action of noise has been analyzed by stochastic averaging and homogenization approaches.[180] Despite these advances, a number of problems on multiple timescale dynamics remain open, especially concerning the non-autonomous systems, the impact of high dimensionality of the fast flow, and the scenarios with higher-codimension bifurcations of the fast flow.

Addressing some of these problems, like a high dimensionality of the fast flow in dynamical networks, may require combining other reduction techniques with singular perturbation theory.

This Focus Issue provides two important insights into the theory of multiple timescale dynamics: one concerning non-autonomous systems with a higher co-dimension bifurcation of the fast subsystem, and the other related to developing the concepts of slow and fast chaos. On the former, Xia *et al.* employ the slow–fast reduction to a system of two coupled Bonhoeffer–van der Pol oscillators with a slow periodic drive, demonstrating how the higher co-dimension bifurcation of the fast subsystem affects the type and stability of bursting solutions.[39] It turns out that the co-dimension two double-Hopf bifurcation, where two stable limit cycles are created, gives rise not only to single-mode but also to two-mode bursting solutions with two types of fast oscillations. Such solutions are shown to even stably coexist for certain conditions, resulting in bursting multistability. It further explains how the features of the slow periodic drive may be tuned to control the switching between the different bursting regimes.

Jaquette *et al.* refine chaos theory in relation to systems with multiple timescale dynamics, arguing for the existence of *slow chaos*.[181] The latter refers to chaotic attractors that display irregular fluctuations at fast timescales but, in contrast to fast chaos, maintain macroscopic regularity and robustness to perturbation at slow timescales. This new paradigm is deemed to be important for biological systems, such as central pattern generators, cardiac cells, or the brain, where chaos has to be tamed to maintain homeostasis at physiologically relevant timescales. Slow chaos is shown to exist only for finite scale separation. Moreover, it is demonstrated that the universal scenario of transition from slow to fast chaos involves a slow passage of relaxation cycles through a crisis of chaotic attractor, underlining the role of its ghost in the emergence of fast chaos.

IV. TIME-VARYING DYNAMICAL NETWORKS: BLINKING AND ADAPTATION

Dynamical networks, where nodes are dynamical systems and links represent their interactions, form the foundation for studying complexity. In many applications, the structure of dynamical networks—including both links and nodal parameters—is not time-invariant, i.e., static, but evolves over time. These time-varying (alternatively evolving or temporal) networks offer a more accurate representation of real-world systems.[61,169,182,183] The changes in networks' structure may arise for various reasons, such as adaptation, external influences, and forcing, or the nodes may be occupied by mobile agents with interactions sensitive to spatial proximity. In general, the evolution of time-varying networks may or may not depend on the dynamical states of nodes. In this respect, they can broadly be cast into two classes: state-independent and adaptive (state-dependent) networks. Both classes usually involve multiscale dynamics due to the separation of the characteristic timescales of coupling and nodal dynamics.

A. Regime switching in blinking networks

The most often considered example of state-independent networks is blinking networks,[182,184,185] where the interactions between the nodes are stochastically switched on or off, with the switching process typically considered to be fast compared to the timescale of nodal dynamics. A more recent realization of blinking networks concerns the scenario where the links are always on, but the form of coupling function is switching.[186] Other examples of state-independent networks include agent-based models,[187] like temporal proximity graphs,[169] where the interactions evolve depending on the current spatial locations of the agents, and metapopulation models,[188] where the nodes are occupied by populations of agents capable of migrating between the populations. For the typical scenario of on–off fast blinking networks, it has been shown that their long-term dynamics generally converges to that of an average network with time-independent connectivity.[184,185] Exceptions may arise if the average network is multistable or if its attractors are not invariant under the switching process.[185]

The central topics in the study of blinking networks have so far been the stability of the synchronization manifold and the possibilities of enhancing synchronization by fast blinking networks.[169] Apart from the stability criterion for the fast blinking case,[189] it has surprisingly been shown that there may also exist separate domains of intermediate switching rates, called "windows of opportunity,"[62] which support stable synchronization. In terms of synchronization control, the MSF approach has revealed several scenarios where fast blinking networks with pairwise,[190] or higher-order interactions[191] may promote synchronization compared to time-invariant networks.

In this Focus Issue, the topic of state-independent time-varying networks has been addressed in four papers considering the issues of convergence to the dynamics of an average network, as well as control of synchronization and extreme events. In particular, Dayani *et al.* employ the MSF approach (cf. Sec. III C) to construct the optimal time-varying coupling configuration to enhance synchronization in complex networks of oscillators.[65] Their method reduces the synchronization threshold and improves the convergence rate toward synchronization, increasing its robustness, as illustrated for coupled Rössler, Chen, and Chua chaotic oscillators.

Sivaganesh *et al.* examine how establishing synchronization and enhancing its stability may be achieved by transient and optimal uncoupling in unidirectional networks of non-identical counter-rotating chaotic oscillators.[66] MSF approach, considered in Sec. III C, is employed to corroborate the effectiveness of the suggested method, illustrating it for the coupled Rössler and Sprott oscillators.

Sriram *et al.* consider an example of a blinking system where a multistable laser undergoes parametric perturbation that involves fast periodic switching between two parameter values.[192] It is shown that such a system may not converge to the average dynamics. In particular, due to the multistability of the unperturbed system, the dynamics of the blinking system depends on the switching parameters and converges to the average attractor only if the two-parameter switching values lie close to the average. Conversely, for distant switching parameters, the blinking attractors may be multistable and substantially different from the average attractor.

Kingston *et al.* consider the emergence of recurring extreme events in systems of coupled oscillators where time-dependent interactions determine the duration and frequency of interactions.[67] Blinking couplings are modeled by a periodic step function, whereby within the period, they switch between two levels of different durations. Control strategies introduced to suppress the onset of extreme events are considered in Sec. VI D.

B. Regime switching in adaptive networks

The paradigm of adaptive networks has become pervasive in various fields, from neuroscience (synaptic[193,194] and homeostatic plasticity,[195] adaptive myelination[196]) and transport networks[197] over cooperative social behavior[198,199] and epidemic spreading,[200] to more recent developments concerning deep learning,[201] reservoir computing,[202] physiological networks modeling cancer genesis and sepsis,[203] and analogies to power grid models.[204] Adaptation is classically characterized as a co-evolution of the nodal and links' dynamics, whose interdependence may be seen as feedback between the networks' structure and function (see Refs. 61 and 183) for a comprehensive review. Nevertheless, certain types of adaptation may be "kinetic," such that the feedback between the collective and nodal dynamics leads to changes in nodal dynamics due to variation in the mean-field signal, as e.g., in cases of frequency adaptation in clapping audiences[205] or modulation of neuronal excitability due to redistribution of metabolic resources.[116,206–208] The adaptation process may proceed as a sequence of discrete events or may unfold continuously over time,[61] whereby the adaptation rate plays a nontrivial role.[183] The two classically invoked limits concern slow adaptation, where the slowly evolving links may be regarded as parameters for the fast nodal dynamics, or fast adaptation, where the slow–fast landscape is reversed.

Adaptation has often been considered in the context of synchronization, especially in networks of coupled phase or neural oscillators, where it has been shown to strongly enhance multistability and to qualitatively impact the synchronization and desynchronization transitions. In particular, adaptation has been found to give rise to several types of first-order synchronization transitions, including explosive synchronization[209] and single- or multistep transitions resembling heterogeneous nucleation.[210,211] Adaptation may also induce complex desynchronization transitions with a high degree of multistability between the different states of partial synchrony,[167,212–214] such as multi-frequency-cluster states, solitary states, and chimeras. Adaptive networks are also used as a tool in control theory (see Sec. VI D), especially in terms of stabilization of the fully synchronized state and increasing its basin of attraction[215] via adaptive coupling weights (gain control)[216] or rewiring of network topology (edge snapping control).[217] While the relation between adaptation and noise has also gained attention in the context of control, e.g., concerning control of coherence resonance or stochastic bursting,[218] it has also been realized that plasticity in neuronal systems may counteract the impact of noise.[219] As described in Sec. IV B, adaptation has quite recently been highlighted as an important ingredient in the onset of sequential activity patterns.[18,22,124,125]

In this Focus Issue, the role of adaptation has been considered in light of applications to epidemic spreading, preservation of biodiversity in ecosystems, reservoir computing and the impact of ketogenic diet in suppressing excessive synchronization associated with epileptic seizures. In particular, Clauß and Kuehn[68] consider epidemic spreading in random and scale-free networks with co-evolutionary dynamics, where the node states change due to epidemics and the network topology evolves via the creation and deletion of edges. The epidemic process involves self-adaptive dynamics, where the switching between adaptation strategies depends on the state and the history of the epidemic system. The key result concerns the emergence of oscillations for the simple setup involving the threshold base application of lockdown measures. The period of oscillations is estimated for random and scale-free networks (cf. Sec. VI D for potential control mechanisms).

Maslennikov *et al.* consider how recurrent networks of rate neurons can be trained within the framework of reservoir computing to learn complex patterns of partial synchrony.[49] The target patterns to be generated and forecasted (cf. Sec. V C) include chimeras, multi-cluster states, and traveling waves, often encountered in ensembles of adaptively coupled phase oscillators. The mechanism of pattern generation is explained at the level of changes in the structure of output weights and the microscopic dynamics of neurons within the reservoir.

Biswas and Ghosh employ game theory to derive the conditions for the emergence of evolutionary stable strategies that give rise to sustainable dynamics of ecosystems.[69] The latter feature population cycles or stationary solutions, which maintain biodiversity. It is shown that evolutionary adaptation is required to suppress the Allee effect, which per se yields a high risk of population extinction (see also Sec. VI D).

Eydam *et al.* consider the impact of metabolic feedback related to a ketogenic diet on the collective dynamics of a heterogeneous, globally coupled population of excitable or tonic spiking excitatory quadratic integrate-and-fire neurons[37] (cf. Sec. III B). The feedback influences the neurons via ATP-dependent hyperpolarizing potassium currents which are shown to trigger a form of kinetic adaptation mechanism, where the excitable units closest to the bifurcation threshold are driven away from it while the frequency of spiking units is reduced. At the collective level, such an adaptation induces new types of multistability between synchronous and asynchronous states, which has ultimately allowed for the development of new strategies to control the switching between the different collective regimes (see Sec. VI B).

V. DATA-DRIVEN APPROACHES FOR PREDICTING REGIME SWITCHING

To understand the dynamics of real complex systems as they evolve over time, we rely on measured datasets. The rate at which we collect data on these systems is rapidly increasing, allowing for deeper insights into their complex behaviors. However, predicting regime switching, such as critical transitions, remains a major challenge, even in systems where the governing equations are known. This difficulty is compounded in data-driven contexts, where the equations are often unknown or incomplete. The existing techniques often require reduction theorems for simplification while facing issues such as noise and irregular time sampling, which complicate the analysis. Specifically, regime switching presents a significant challenge due to the high dimensionality of data in many real-world systems, which frequently necessitates dimensionality reduction for both computational efficiency and interpretation. Noise in the data can obscure underlying dynamics, leading to false conclusions, while irregular sampling times hinder time-series analysis, as many time-series analysis techniques assume uniform data collection intervals. Addressing these issues remains an important focus for the development of more robust and accurate models for complex systems.[43]

In response, modern methods for analyzing datasets associated with regime switching have evolved. These techniques are broadly classified into univariate and multivariate approaches. Univariate methods focus on tracking the evolution of a single observable, while multivariate techniques provide insights into the interactions between different variables, allowing for a more comprehensive understanding of system dynamics and correlations. Moreover, these methods can be further divided into model-free and model-based approaches. Model-free techniques aim to quantify similarity or distance between states of the system,[220,221] while model-based approaches attempt to infer or construct underlying models from the data to describe and predict the system's behavior.[222–225]

Several notable methodologies have been developed for analyzing switching dynamics, including eigenvalue spectrum-based methods, critical slowing down detection, and dynamic network markers. Eigenvalue spectrum-based methods leverage the spectral properties of governing matrices, such as the adjacency or Laplacian matrices, to assess stability, identify critical points, and detect transitions in the system.[226,227] Critical slowing down detection focuses on tracking a system's recovery rate from perturbations, which gradually decreases as the system approaches a critical transition, serving as a reliable early-warning indicator for tipping points.[91,228] Meanwhile, dynamic network markers, which monitor changes in network features like synchronization and connectivity patterns, have proven effective in identifying regime shifts within dynamical networks.[229,230]

Additionally, machine-learning-based methods have gained popularity for their ability to analyze complex, high-dimensional data. Approaches based on neural networks—such as long short-term memory (LSTM) networks and reservoir computing—along with regression models, can predict transitions and uncover hidden patterns in time-series data without requiring explicit knowledge of the system's governing equations.[50,231,232] Sparse identification, in particular, provides a straightforward way to capture relationships between variables and can be effective for forecasting transitions based on historical data. These data-driven techniques have shown promising results in predicting critical transitions in systems characterized by nonlinear and even chaotic dynamics.

This Focus Issue delves into data-driven approaches for comprehending switching dynamics by integrating dynamical systems theory with statistical learning techniques.[45–49] Through this integration, the aim is to improve our ability to detect and manage critical transitions in complex systems, particularly in cases where analytical techniques may fall short due to lacking governing equations or models, the system's complexity, or data constraints.

A. Sparse identification

Sparse identification methods aim to reconstruct dynamical systems using a library of candidate functions through techniques, such as sparse regression[233] or mutual information-based approaches like causation entropy.[234] These methods quantify the contributions of chosen candidate functions to model the dynamics represented by the data. The reconstructed models can then be used to identify regime-switching events. A key challenge in this field is reconstructing the dynamics in real-time, known as online learning.[235] It requires analyzing short data batches simultaneously to build the dynamical model and predict regime switching.

In this Focus Issue, the CEBoosting online learning algorithm leverages causation entropy to determine the contributions of candidate functions in reconstructing dynamics and identifying regime shifts.[46] Despite the brevity of each data batch, the accumulated causation entropy value over a sequence of batches provides a robust indicator. The CEBoosting method is tested on a nonlinear model that simulates the interaction of topographic mean flow. This application showcases the method's capability to detect regime switching online, even in the presence of strong intermittency and extreme events.

B. Functional networks

The functional network approach involves reconstructing a connectivity matrix using similarity measures such as correlation or mutual information. The resulting matrices describe the functional connectivity between system nodes and can be transformed into a network topology by thresholding the matrix elements. When applied to consecutive data windows, changes in the connectivity matrix can indicate regime switching.

This Focus Issue introduces a recent method called "window thresholding," which has been developed to identify characteristic sub-networks with connection strengths within a specific range.[47] By adjusting the window size from smaller to larger values, the transformation or switching of sub-networks across different connection scales can be examined. Window thresholding provides a detailed analysis of brain networks from functional magnetic resonance imaging data, enabling the identification of network components at various connection strength levels. This study investigates how transitions in the structure of functional neural network subnetworks at different connection strengths could serve as biomarkers for diagnosing major depressive disorder. It is shown that in healthy individuals, functional neural networks transition from a combination of scale-free and random topology to small-world networks as connection strength increases. In contrast, patients with the disorder exhibit uncertainty at low connection strengths. To address inter-subject variability in clinical data analysis, a consensus-network approach is introduced.

C. Machine learning

Predicting future system states from past observations is a key research question at the intersection of nonlinear dynamics and machine learning, with neural networks being the primary tools for this task. While many studies focus on optimizing hyperparameters of neural networks such as network size, spectral radius, and neural time constants, there is often less emphasis on understanding the microscopic dynamics and structural changes within networks during training. This gap leaves a limited understanding of how these internal dynamics influence the observed patterns at the functional level. This Focus Issue addresses this by investigating target patterns produced by adaptive networks of phase oscillators, which can exhibit various intriguing regimes through small adjustments in control parameters.[49] It begins by describing these adaptive networks and demonstrating how recurrent neural networks can be trained to generate these patterns autonomously. The exploration continues with an analysis of how specific dynamic and structural features contribute to forming desired multidimensional signals. Finally, the study explains how particular regimes emerge from the interplay between structure and dynamics.

Machine learning methods can also be used to identify potential dynamical instabilities in systems undergoing structural perturbations, such as adding or removing their components. Adding a new component to improve the stability of network dynamics may seem straightforward, but this modification can lead to failures, a phenomenon known as Braess's paradox.[236,237] This unexpected change can easily trigger regime switching and is generally difficult to foresee, potentially causing severe damage. For example, adding a transmission line in power grids can alter power flow distribution and lead to capacity-load mismatches. Identifying potential sources of this paradox in complex power grids has been a long-standing challenge. In this Focus Issue, a deep learning method based on graph neural networks, specifically utilizing graph isomorphism networks, has been developed to predict and avoid Braess's paradox, preventing performance deterioration and instabilities.[48] The research demonstrates the method's efficiency using IEEE standard test cases.

D. Recurrence analysis

Recurrence-based data analysis techniques are well-established and widely used across various research areas.[238] One of the most popular tools in this category, the recurrence plot, reconstructs a multidimensional phase space through the time embedding of scalar time series. This method, however, presents challenges, such as determining the correct embedding dimension and the necessary time delay for reconstruction. While powerful for low-dimensional time-series analysis, these techniques are computationally intensive for analyzing flow dynamics.

In this Focus Issue, a more efficient recurrence-based approach has been introduced to analyze multiphase flow dynamics by examining the angular separation between appropriately defined state vectors.[45] This method is applied to experimental multiphase flows in a bubble column reactor and effectively detects a transition from a regular state, marked by repetitive flow patterns, to a complex dynamic state, characterized by variability in flow patterns over time and space. For temporal analysis, each snapshot of the weighted matrix is converted into a one-dimensional vector, and the angle between pairwise vectors at different time points is calculated. For spatial analysis, vectors are formed from specific locations across consecutive time instants, and the angles between these vectors are determined. The method is shown to have certain advantages, such as requiring less data, compared to classical recurrence plot methods, making it suitable for online detection of rapid regime switching.

E. Topological data analysis

Topological data analysis (TDA) provides a valuable mathematical framework for examining the shape and structure of complex, high-dimensional datasets.[239] It has proven effective in dynamic systems for early-warning detection, and in this Focus Issue, TDA has been applied to the challenge of predicting lean blowout in combustion systems.

In this Focus Issue, TDA has been used for real-time lean blowout prediction across multiple combustor configurations.[44] By leveraging tools from mathematical topology and computer science, TDA uncovers the underlying structure of data, tracking the persistence of key features like connected components and holes in phase-space-embedded time series. The study demonstrates how point summary metrics derived from TDA can effectively capture critical transitions in turbulent systems, offering a robust tool for early-warning detection in both single-burner and multi-burner combustors.

VI. NONLOCAL STABILITY AND CONTROL OF REGIME SWITCHING

The overarching objective of control in complex systems is to stabilize the desired dynamics, enhancing its resilience to external perturbations, such as shocks or noise. Classical control approaches, including feedback, non-feedback, stochastic, and adaptive techniques, have proven quite effective in managing certain problems, like control of chaos[240,241] or synchronization.[242,243] However, they may still be challenged by the inaccessibility of certain degrees of freedom, or the presence of high multistability and the need to account for global state space properties. In light of increasingly demanding applications, there is a growing need for both generic and specific schemes to induce or suppress instances of regime switching, affect the preference of attractors by adjusting the state space properties,[55] or maintain the system dynamics in the vicinity of certain transitions, like the "edge of chaos"[58,59] or the "edge of criticality,"[56,57] to achieve optimal performance.

Recent advances in data-driven approaches, particularly machine learning and model-free prediction, have further enhanced our ability to manage complex dynamics by enabling the detection and prediction of critical transitions. These methods have allowed for the timely intervention in networked systems, where cascades of regime shifts can lead to catastrophic outcomes. The combination of classical and modern control methods, alongside data-driven approaches, highlights the increasing versatility of control strategies that are being applied across a wide range of disciplines, as illustrated by this Focus Issue.

A. Feedback control

Feedback control, one of the most ubiquitous control strategies, is a closed-loop scheme that leverages the system's internal state to deliver it back into the system, either instantaneously or with a delay.[244] Delayed feedback is classically deployed in the context of Pyragas chaos control to stabilize unstable periodic orbits embedded in chaotic attractors.[241] On the other hand, instantaneous feedback is typically harnessed in multistable systems to stabilize the selected state against stochastic perturbations, thus preventing the undesired attractor hopping.[55] In the presence of stronger noise, this can efficiently be achieved via reinforcement learning,[245] which may solely be based on data in a model-free scenario. In the case when the target state is complete synchronization in large networks, applying feedback to all the nodes may not be feasible, which can be resolved by the so-called pinning technique.[243] The latter involves implementing the feedback to just a small subset of nodes, which are either preselected, if a detailed knowledge of the network topology is available, or are changing adaptively by a time-varying controller.

In this Focus Issue, feedback control is explored in relation to the transition between the regular and chaotic behavior. In particular, Öztürk *et al.*[63] consider the impact of a delayed feedback in the context of a minimal gene regulatory circuit capable of displaying chaotic behavior. Inspired by synthetic gene networks, an extended model of a two-gene regulatory circuit with delayed bidirectional interactions is introduced, demonstrating that chaos can emerge without inhibitory delayed self-feedback. While biological gene networks are typically not chaotic, they are believed to be capable of operating at the edge of chaos or criticality, similar to neural networks.[58,246] The reported findings suggest that delayed couplings could be an important ingredient for controlling gene regulatory circuits, possibly allowing them to operate in the vicinity of the transition to chaos.

B. Non-feedback control

In addition to closed-loop feedback control, there are state-agnostic approaches to control that do not require the direct measurement of the system state. Such non-feedback, or open-loop control schemes, may involve the implementation of short external pulses, or perturbations to the systems' parameters or variables. Examples include the application of noise to improve regularity of oscillations via coherence resonance,[247] the use of parametric perturbations to control the onset of oscillations[248] or slow harmonic perturbations to parameters or state variables to eliminate system attractors,[55] as well as impulse control through sudden jumps in state variables.[249]

In this Focus Issue, non-feedback control strategies are considered for managing synchronous neural activity, a central topic in the research on human epilepsy disorder. Eydam *et al.* present a theoretical framework to address this issue by focusing on manipulating the metabolic feedback associated with ketogenic diet.[37] The proposed key mechanism for dietary control of neural activity relies on activating the ATP-sensitive potassium channels. Using the results of bifurcation analysis of the corresponding mean-field model, cf. Sec. III B, three control strategies to trigger transitions from epileptic to healthy dynamical regimes are developed. The latter include parametric perturbation by adjusting the ATP production rate, the shock-like dynamic perturbation induced by sudden changes in the ATP concentration, and the application of external pulse currents.

C. Nonlocal stability and stochastic sensitivity analysis

The ability to control a system's dynamics is intricately linked to the knowledge of the state's stability. While most commonly, local and linear stability conditions are considered, many applications in complex systems require nonlinear effects, global bifurcations, and nonlocal state space properties to be considered to deliver robust control schemes. One successful technique to incorporate nonlocal aspects of the phase space is the so-called basin stability analysis, which provides a global and probabilistic perspective on a system's resilience to perturbations.[250,251]

For power grids, control strategies are crucial to achieve optimal operation. Khramenkov *et al.* propose a method for switching between coexisting operating modes to prevent short-term outages or blackouts and maintain stability in the presence of temporary perturbations or noise.[71] A grid model featuring three synchronous generators supplying a common static load is shown to exhibit homogeneous (symmetric) and inhomogeneous (asymmetric) steady states, as well as librational (quasi-synchronous states) and rotational limit cycles and chaotic attractors (asynchronous modes). The nonlocal stability of regimes is probed by basin stability analysis to investigate the impact of strong and realistic perturbations. For the parameter domains supporting high multistability, an attractor preference method to switch to a selected optimal state is presented.

Nicolaou and Bramburger consider the mechanisms of onset and disappearance of stationary and traveling periodic localized patterns in systems of coupled oscillators, focusing on the examples of symmetry-breaking chimeras in rings of Janus oscillators and gap solitons in arrays of parametrically driven pendula. In contrast to classical snaking bifurcations giving rise to localized steady states, the authors demonstrate new bifurcation scenarios of symmetry-breaking localization involving heteroclinic cycles and non-attracting chaotic invariant sets.[64]

In a metapopulation model of two oscillator populations coupled by migration, Ryashko *et al.* consider the transitions triggered by the noise in the migration intensity.[70] The switching unfolds between anti- and in-phase synchronization modes, as well as quasi-regular and chaotic metastable regimes. By combining numerical simulations to study attractor basins with a stochastic sensitivity approach to probe the robustness to noise, they showcase the importance of transient chaotic dynamics for the switching process. It is also demonstrated how switching can be predicted by taking into account the stochastic sensitivity of attractors and their basins' geometry.

D. Control by time-varying networks

The concepts of time-varying networks have emerged as a fundamental paradigm in many research fields related to complexity, including neural and brain networks,[252] power grids,[253] and epidemic spreading[254] (see Sec. IV B for more details). Adaptive and blinking networks are increasingly emerging as an important tool in control theory, being often invoked in the context of stabilizing the fully synchronized state. In that regard, several strategies for optimizing the blinking process,[190,191] as well as adaptive techniques have been developed, including the continuous adaptive control,[215] edge snapping,[217] and the speed-gradient method.[61] Adaptation is also considered as a means to control stochastic phenomena, including coherence resonance[218] and noise-induced switching.[255] Nevertheless, control of adaptive networks per se is also gaining attention, especially given their ubiquity in biological systems, from transportation networks to the genome and the brain.[243] In this Focus Issue, several contributions highlight the application of time-varying networks to control of complex systems' dynamics.

In epidemic modeling, Clauß and Kuehn[68] incorporate the concept of self-adaptive dynamics, where the switching in strategy space depends on the history of the epidemic process (cf. Sec. IV B). It is indicated that introducing an observable that controls the switching mechanism by affecting the infection numbers may present one of the simplest mechanisms to obtain self-organized criticality in epidemic dynamics.

For complex oscillator networks, Dayani *et al.*[65] use the MSF approach to construct an optimal time-varying coupling configuration that enhances the synchronization process (see Sec. IV A for more details). The optimization is reflected in reducing the cost of synchronization by improving the convergence rate and decreasing the critical threshold. The developed control method is indicated to be applicable to arbitrary network topology.

Kingston *et al.*[67] develop a method for suppressing the onset of extreme events in blinking networks of coupled oscillators (also see Sec. IV A). The control strategy consists in shifting the transition point between the regular and extreme events by adapting the duration of interaction between system elements. The method is illustrated for small systems and networks of coupled FitzHugh–Nagumo and Líenard oscillators but is indicated to be applicable for a wide range of oscillator networks.

Biswas and Ghosh apply game theory to introduce an evolutionary adaptation strategy to overcome the Allee effect[69] (see Sec. IV B for more details). Such adaptive control is associated with a trade-off, as the increase in the grazing burden of predator comes at the expense of its accelerated mortality rate. Evolutionary stable strategies, intended to support biodiversity, are formulated in such a way that mutant counterparts cannot invade original populations. These strategies are established to relate to local optima of certain functions of model parameters.

VII. CONCLUSION

Regime switching refers to transitions observed in complex systems between the qualitatively different states which may be asymptotically stable or may conform to long transients. These transitions can occur due to internal dynamics, external perturbations, or changes in system parameters, making regime switching a central concept for understanding and predicting the behavior of natural and engineered systems. Since the 2018 Focus Issue of Chaos on multistability and tipping,[72] the field of regime switching has expanded in multiple directions, witnessing the emergence of new paradigms, both in terms of focal problems and methodology. The concept of tipping has evolved to encompass more gradual and reversible changes,[11–13,82] and classifications now also include events triggered by sudden, shock-like perturbations.[73] Substantial interest has also gained sequential and cyclic forms of regime switching, including tipping cascades,[14,83,256] emergent phenomena such as heteroclinic cycles and networks,[20,23,104,105] noise-induced switching between highly structured metastable states,[257,258] and cyclic patterns of partial synchronization facilitated by a slow–fast decomposition in adaptive networks.[18,22,124,125] These developments underline the need to broaden the notion of regime switching to transitions not only between stable states but also between long transients, such as quasi-attractors or metastable and pseudo-stationary states. Further new insights have emerged from the study of multiple timescale dynamics[38] and the role of nonlocal stability,[79] with a particular focus on chaotic saddle structures.[73] These advances reflect the growing complexity of regime-switching phenomena across diverse systems.

In terms of theory, the expansion of the field has been facilitated by the arrival of powerful rigorous reduction approaches. The frameworks derived from Ott–Antonsen (OA)[28,29] and Montbrió–Pazó–Roxin (MPR)[32] methods have closed the gap between the apparent high dimensionality of phase spaces and the effective low-dimensionality of collective dynamics in coupled oscillator and neuronal systems, respectively, prompting the efficient use of bifurcation theory in the study of multistability and regime switching. Other reduction techniques, such as the master stability function approach,[26] have been adjusted to accommodate for the time-varying structures, including blinking[190] and adaptive networks.[167,204] In parallel, the research on improving the approximate methods of dimensionality reduction[127,174] for the analysis of tipping in networks is also gaining momentum.

The progress in data-driven approaches has significantly enhanced our ability to detect and predict complex regime switching from time-series data. These methods, supported by the recently emerged efficient deep learning frameworks,[50–53] are providing fresh insights into critical transitions and form an important part of this Focus Issue. The control theory of regime switching has also seen considerable advances. Non-feedback, stochastic, and adaptive control strategies are emerging as efficient and practical alternatives to traditional feedback control, especially in the face of the latter's implementation challenges. These developments form another key theme of the Focus Issue, demonstrating that theoretical and applied methods evolve in parallel to contribute to the study of regime switching in complex systems.

In light of the stated above, the main advances brought by this Focus Issue can be cast into three groups: reduction approaches, data-driven detection of regime switching, and control theory. In terms of the development and application of reduction approaches, the most important contributions include (i) the OA and MPR formalisms, (ii) multiple timescale analysis, and (iii) dimension reduction techniques. With regard to (i), we underline (a) the extensions of the OA and MPR formalisms to populations of oscillators[34] and neurons[33] subjected to local non-Gaussian white noise; (b) the exploration of finite-size effects in triggering noise-induced switching between coexisting metastable states, including those associated with periodic and chaotic attractors from the thermodynamic limit;[35,36] and (c) the role of metabolic feedback via ketogenic diets in triggering new multistability forms and hysteretic transitions between asynchronous and synchronous dynamics in neuronal populations.[37] For (ii), we highlight the use of multiple timescale analysis in (a) explaining multimode bursting in coupled forced systems with a co-dimension two bifurcation of the fast subsystem,[39] and (b) its role in distinguishing slow and fast chaos.[181] In terms of (iii), the dimension reduction approach has been applied to determine the critical point and the local structure of epidemic spreading regimes in networks with higher-order interactions.[177]

The important problems of nonlocal stability analysis and transients associated with chaotic saddles have been considered in the context of pattern formation and transitions between coexisting regular and chaotic regimes. Concerning the former, new scenarios involving heteroclinic cycles and saddle chaotic sets have been reported for the onset of stationary localized patterns in coupled oscillators.[64] In the latter context, the basin stability analysis has been employed to optimize the operation of power grid models in the presence of high multistability,[71] while the stochastic sensitivity approach has been applied to explain noise-induced transitions between quasi-regular and chaotic metastable states.[70]

This Focus Issue also contains valuable new insights into data-driven detection of regime switching leveraging (i) sparse identification, (ii) machine learning, (iii) network-based analysis of time series, (iv) recurrence method, and (v) geometric approach to data analysis. Concerning (i), the causation entropy-based CEBoosting online detection algorithm[46] has been introduced, indicated to be robust even in the presence of intermittency and extreme events. Regarding (ii), the reservoir computing framework has been applied to detect the onset of complex patterns of partial synchrony,[49] while deep learning has been employed to detect power grid failures.[48] In the framework of (iii), functional neural networks have been employed to detect the onset of neurological disorders.[47] In terms of (iv), the recurrence plot method has been adapted to detect the onset of multiphase flow patterns in bubble column reactors in chemical industries.[45] In the context of (v), the topological data analysis method has been developed to detect lean blowout in combustion systems.[44]

Finally, we underline the important advances in control theory in the directions of the application of time-varying networks and the non-feedback control. Regarding the former, the dynamics of blinking networks has been optimized to enhance synchronization between chaotic oscillators[65,66] or to prevent the onset of extreme events,[67] while adaptive strategies have been suggested to control epidemic spreading.[68] In the latter case, the different forms of external pulses, parametric and shock perturbations have been indicated as potential means to suppress epileptic seizure-like dynamics.[37]

Given the reported advances in mathematical approaches and the broad spectrum of applications covered, this Focus Issue offers a unifying perspective on regime switching. We hope this synthesis will catalyze further dialog between various branches of complexity science, particularly the interaction between the fields of tipping and extreme events, as well as applications involving adaptation, reservoir, and neuromorphic computing. We anticipate that novel methodologies, perhaps involving more sophisticated prediction algorithms, new hybrid models or methods for control of sequential activity, will emerge from these interactions.

Additional focus on extending bifurcation theory to nonautonomous systems could provide a more robust framework for analyzing time-varying and driven systems, which is essential for understanding complex real-world phenomena. Furthermore, a deeper comprehension of multiple timescale dynamics, especially when the fast subsystem exhibits high dimensionality or higher-codimension bifurcations, is critical. In the latter context, it seems likely that different applications will increasingly require the combined use of multiple reduction techniques.

Recent advances in data-driven approaches have significantly enhanced our ability to detect and predict regime switching in complex systems, particularly critical transitions between different attractors.[54] While avenues for forecasting based solely on pretransition data are open, challenges remain concerning transitions between long-transient dynamics or cascading transitions in networked systems.[259] A promising direction involves extending current methods to account for higher-order interactions between nodes in complex networks.[260] While machine learning models trained on synthetic data show promise for predicting critical transitions in empirical systems, developing model-free prediction techniques offers a significant opportunity for future research. Efforts to diversify synthetic models could improve the robustness of these approaches in real-world examples.[50] Although bifurcation classification methods are available, pinpointing the exact moment of a transition remains elusive. Another ingredient for accurate prediction concerns understanding how random fluctuations, especially non-Gaussian and bounded noise, compete with deterministic forces. Advances in these directions hold significant potential for improving the early detection and control of regime switching in a wide range of applications, from power grids and financial markets to ecosystems, climate, and neuroscience.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Igor Franović: Conceptualization (equal); Investigation (equal); Methodology (equal); Resources (equal); Writing – original draft (equal); Writing – review & editing (equal). **Sebastian Eydam:** Conceptualization (equal); Investigation (equal); Methodology (equal); Resources (equal); Writing – original draft (equal); Writing – review & editing (equal). **Deniz Eroglu:** Conceptualization (equal); Investigation (equal); Methodology (equal); Resources (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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