Nonlinear BEC Dynamics Induced by the Harmonic Modulation of the Atomic *s*-wave Scattering Length

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Motivation: In the recent experiment [1], nonlinear dynamics of a BEC was induced through the harmonic modulation of the atomic *s*-wave scattering length, and collective oscillation modes were observed. Using perturbation theory and numerical approach, we study frequency shift of the collective mode, due to nonlinear interaction effects [2].

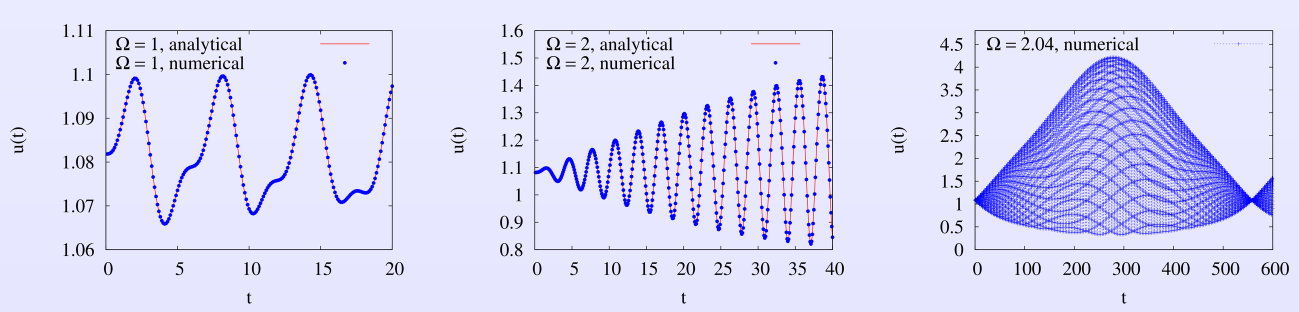
Time-dependent variational description of BEC

- Time-dependent Gross-Pitaevskii equation can be studied using a Gaussian variational ansatz [3]. In this approach, the dimensionless condensate width $u(t)$ evolves according to

$$\ddot{u}(t) + u(t) - \frac{1}{u(t)^3} - \frac{P}{u(t)^4} = 0\,,$$

where $P = \sqrt{\frac{2}{\pi}}\frac{Na}{l}$, a is the *s*-wave scattering length, $l = \sqrt{\frac{\hbar}{m\omega}}$ is the length scale for the harmonic trap, and N is the number of atoms.

- Using Feshbach resonances, harmonic modulation of the scattering length was achieved [1], yielding the time-dependent interaction $P(t) = p + q\sin\Omega t$. The figures below give real-time dynamics of BEC for $p = 0.4$, $q = 0.06$ and different driving frequencies Ω.



Analytical Poincaré-Lindstedt analysis

- Linearization of the variational equation yields zeroth order collective mode $\omega = \omega_0$ of oscillations around the time-independent solution u_0:

$$\omega_0 = \sqrt{1 + \frac{3}{u_0^4} + \frac{4p}{u_0^5}}\,, \qquad u_0 - \frac{p}{u_0^4} - \frac{1}{u_0^3} = 0\,.$$

- To calculate the collective mode to higher orders, we rescale time as $s = \omega t$:

$$\omega^2\,\ddot{u}(s) + u(s) - \frac{1}{u(s)^3} - \frac{p}{u(s)^4} - \frac{q}{u(s)^4}\sin\frac{\Omega s}{\omega} = 0\,.$$

- Far from resonances, we assume the following perturbative expansions in q:

$$u(s) = u_0 + q\,u_1(s) + q^2\,u_2(s) + q^3\,u_3(s) + \ldots\,,$$
$$\omega = \omega_0 + q\,\omega_1 + q^2\,\omega_2 + q^3\,\omega_3 + \ldots$$

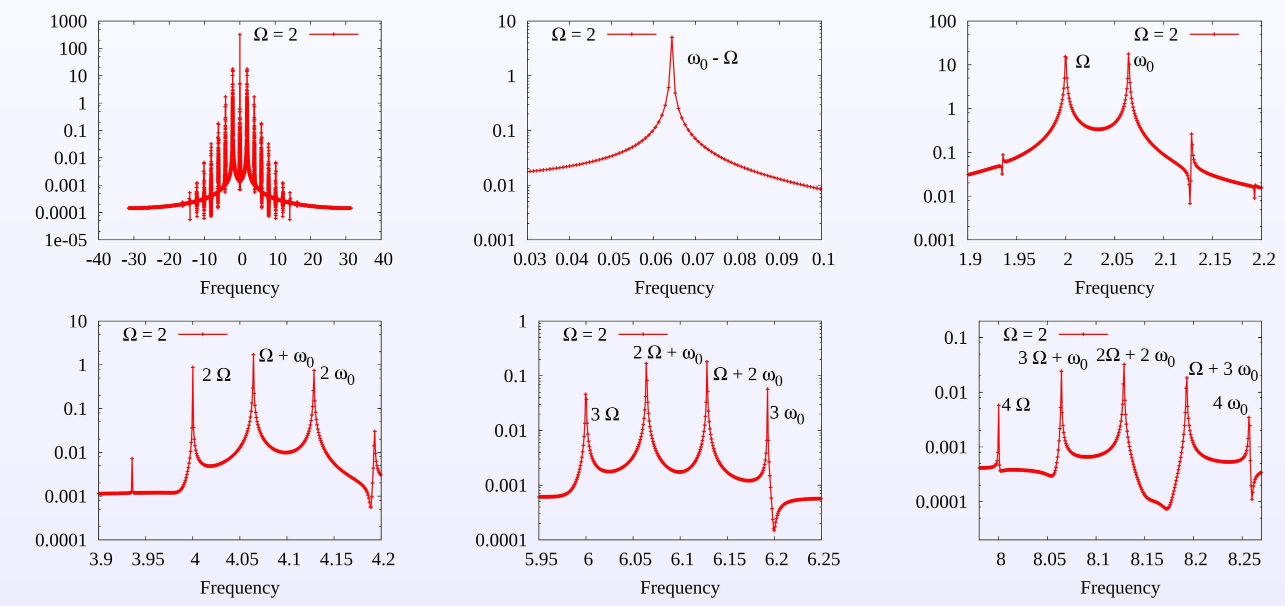
- This leads to a hierarchical system of equations in orders of q [4]:

$$\omega_0^2\,\ddot{u}_1(s) + \omega_0^2\,u_1(s) = \frac{1}{u_0^4}\sin\frac{\Omega s}{\omega}\,,$$
$$\omega_0^2\,\ddot{u}_2(s) + \omega_0^2\,u_2(s) = -2\omega_0\,\omega_1\,\ddot{u}_1(s) - \frac{4}{u_0^5}\,u_1(s)\,\sin\frac{\Omega s}{\omega} + \alpha\,u_1(s)^2\,,$$
$$\omega_0^2\,\ddot{u}_3(s) + \omega_0^2\,u_3(s) = -2\omega_0\,\omega_2\,\ddot{u}_1(s) - 2\beta\,u_1(s)^3 + 2\alpha\,u_1(s)u_2(s) - \omega_1^2\,\ddot{u}_1(s) + \frac{10}{u_0^6}\,u_1(s)^2\,\sin\frac{\Omega s}{\omega} - \frac{4}{u_0^5}\,u_2(s)\,\sin\frac{\Omega s}{\omega} - 2\omega_0\,\omega_1\,\ddot{u}_2(s),$$

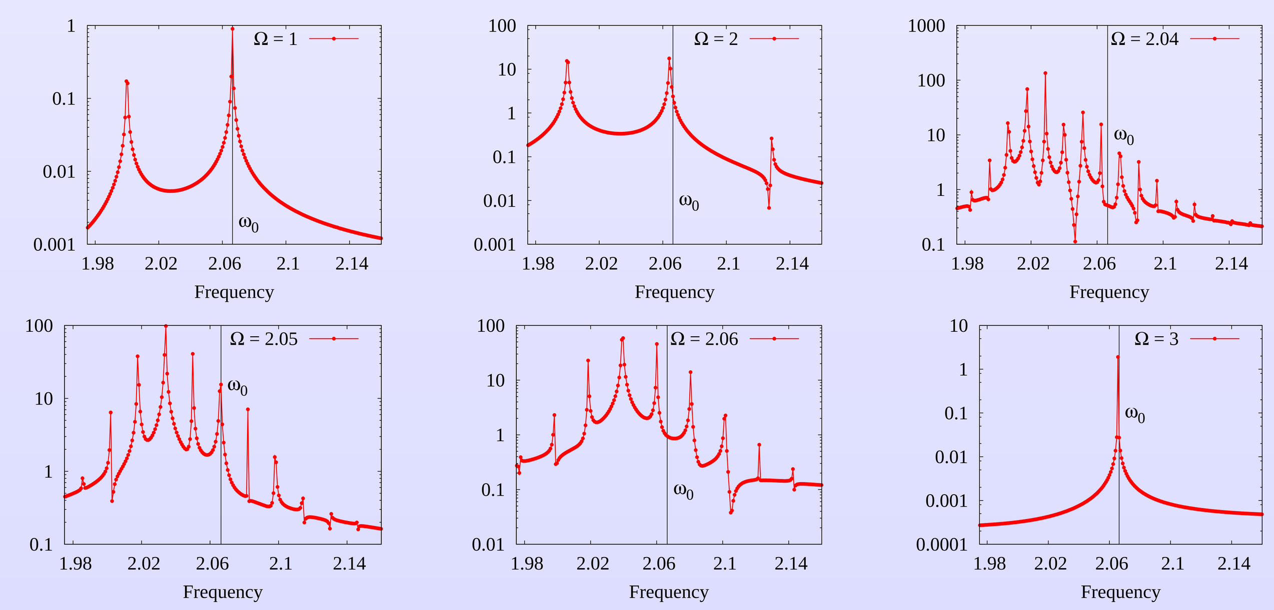
where $\alpha = 10p/u_0^6 + 6/u_0^5$ and $\beta = 10p/u_0^7 + 5/u_0^6$.

Fourier analysis of numerically obtained solutions

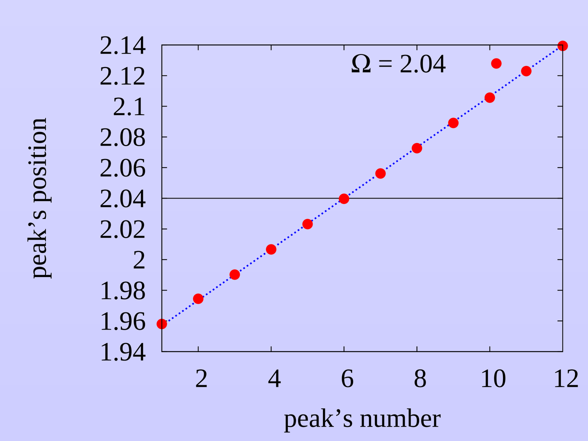
- Numerical solutions for the time interval $(0, T)$ with the time step Δt are analyzed using the discrete Fourier transform. This way, maximal accessible frequency is $\omega_{max} = \pi/\Delta t$, while the resolution is $\Delta\omega = 2\pi/T$.



- The above graphs give the Fourier spectrum of $u(t)$ with basic modes $\omega_0 \approx 2.06638$ and Ω, higher harmonics $n\omega_0$ and $m\Omega$, as well as linear combinations $n\omega_0 + m\Omega$, including the beating frequency $|\Omega - \omega_0|$.
- Resonant effects are expected for $\Omega \approx \omega_0$, as numerical results clearly confirm on the graphs below. However, a complex peak structure close to ω_0 appears, and a shift in the frequency $\omega_0 \to \omega$ is clearly visible.



- For $\Omega \approx \omega_0$, frequencies $\omega(k) = \Omega + k \times |\omega - \Omega|$ where k is a small integer are all close to the resonant region, and are all significantly excited, as we observe numerically.
- Prominent peaks around ω_0 are equidistant, as we see from the graph below for $q = 0.1$. By fitting a linear function $\omega(k) = A + B \times k$ to numerical data, we determine $|\omega - \Omega|$ for each Ω, and calculate collective modes as $\Omega \pm A$, given in the table.



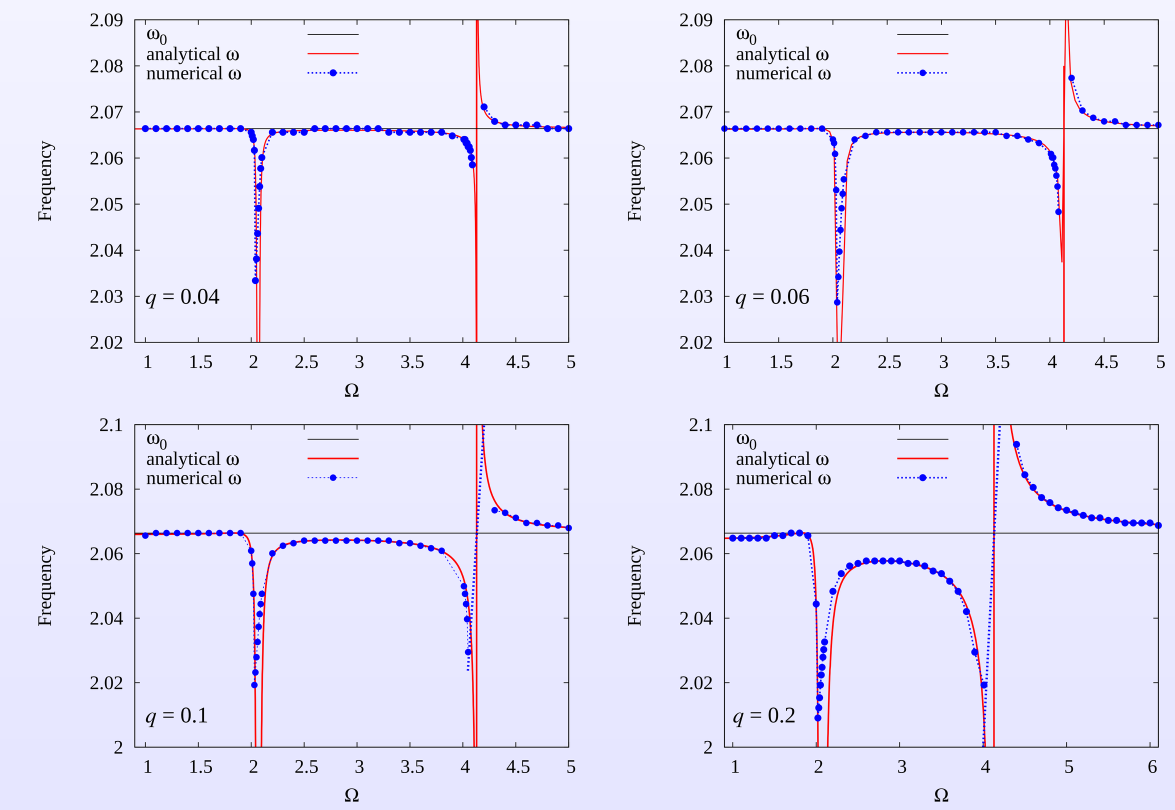
Ω	A	$\Omega - A$	$\Omega + A$
2.00	0.0615	1.9352	**2.0609**
2.04	0.0166	**2.0232**	2.0562
2.05	0.0218	**2.0279**	2.0719
2.06	0.0273	**2.0326**	2.0876

Frequency shift of the collective mode

- Frequency shift of the main collective mode is obtained using the third order Poincaré-Lindstedt method in q. First order correction ω_1 vanishes, leading to the frequency shift quadratic in q:

$$\omega = \omega_0 + q^2\,\frac{Polynomial(\Omega)}{(\Omega^2 - \omega_0^2)^2\,(\Omega^2 - 4\omega_0^2)}\,.$$

- We have obtained good agreement of numerical and analytical results for the frequency shift far from resonances, as can be seen from the graphs below for various values of q.



- The most significant shift of up to 5% is observed for $\Omega \approx \omega_0$ and large q.

Summary and outlook

- Using Fourier analysis of numerical data and analytical Poincaré-Lindstedt method, we calculated shift of the collective mode for a spherically symmetric BEC excited by harmonic modulation of the scattering length.
- In order to compare analytical results with the experiment [1], we are working on a similar perturbation theory for a cylindrically symmetric BEC.
- To further study nonlinear BEC dynamics effects, we will use numerical simulations of the full time-dependent Gross-Pitaevskii equation.

References

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