Resistance in Percolating Networks

Milan Žeželj, Igor Stanković, Aleksandar Belić

Scientific Computing Laboratory, Institute of Physics Belgrade,
University of Belgrade, Serbia

contact: milan.zezelj@scl.rs

SCIENTIFIC COMPUTING LABORATORY

(1) Motivation

Percolation is random process, and we can distinguish two types of the percolation problems: lattice percolation and continuum (or irregular lattice) percolation. It is widely accepted that lattice and continuum percolation belong to the same class in the sense that the latter possesses the same critical exponents as the former.

Stick percolation has not been extensively studied theoretically until now [1,3]. Still it is an important representative of continuum percolation and interesting due to its relevance for systems consisting of conducting rodlike particles.

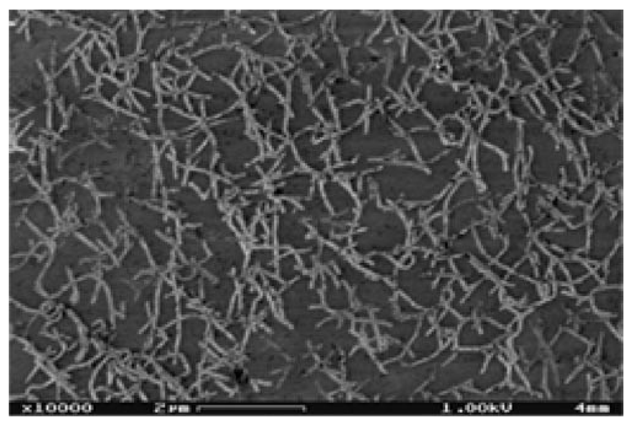
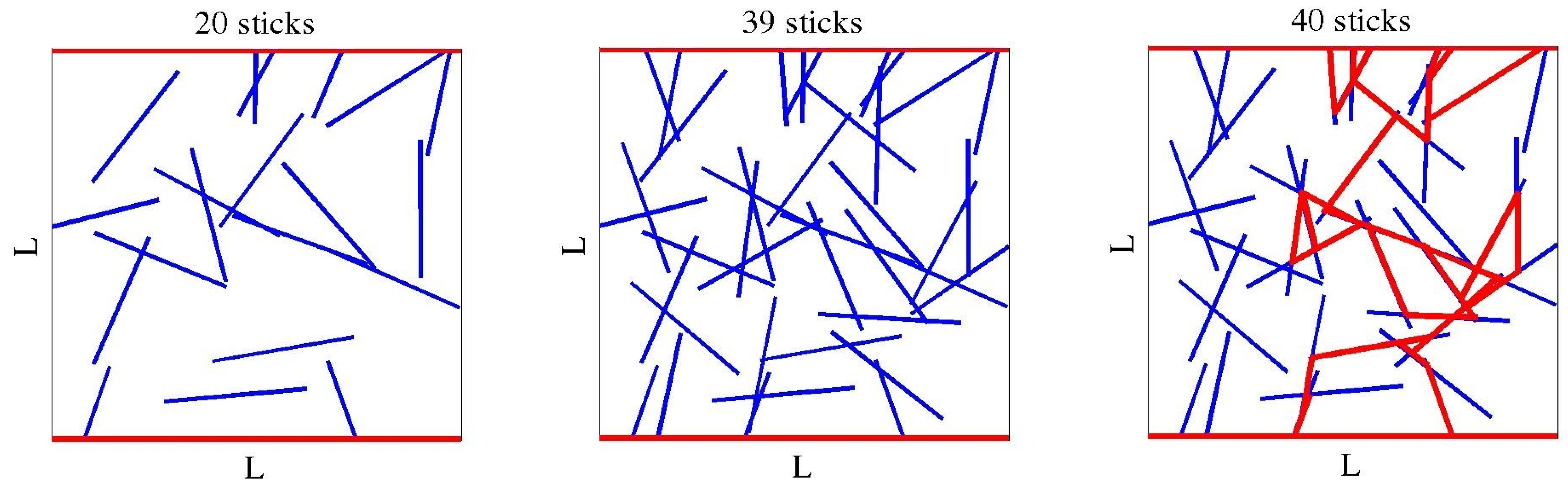


Illustration of carbon nanotube network taken from M.Y. Zavodchikova et. al., Nanotechnology 20 (2009) 085201.

(2) Monte Carlo simulations of rodlike particle percolation

In Monte Carlo simulations the sticks are uniformly placed between two electrodes with homogeneous distribution of stick centres and angles. The resistance of the whole stick of length l is unity, and contact resistance is assumed to be zero. The boundaries are free in horizontal direction and electrodes are connected to top and bottom of the simulation box. At percolation point, there is current flowing through system. We calculate current through the system and determine the conductivity from Kirchhoff's law. System size L is fixed and L/l=10.

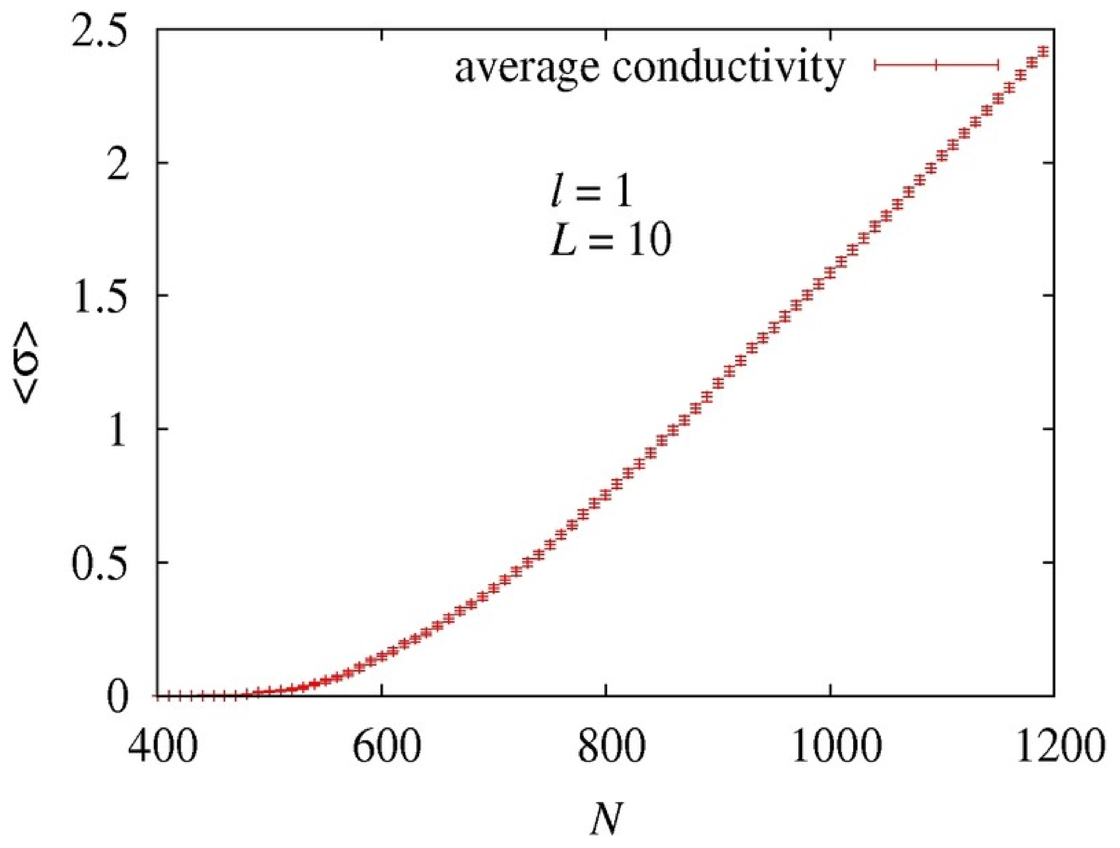


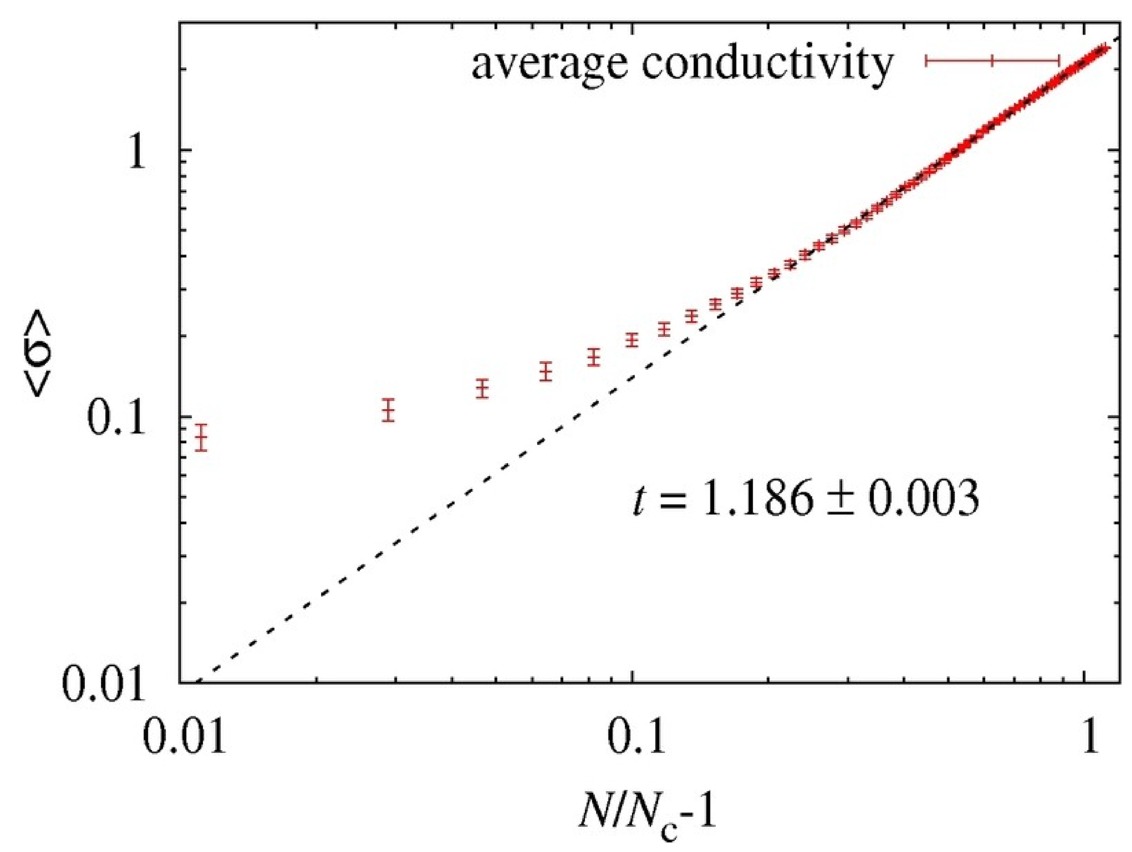
For a general stick length l and a system size length L, the percolation threshold (number of sticks N_c) is:

$$N_c \frac{l^2}{L^2} = 5.6\,.$$

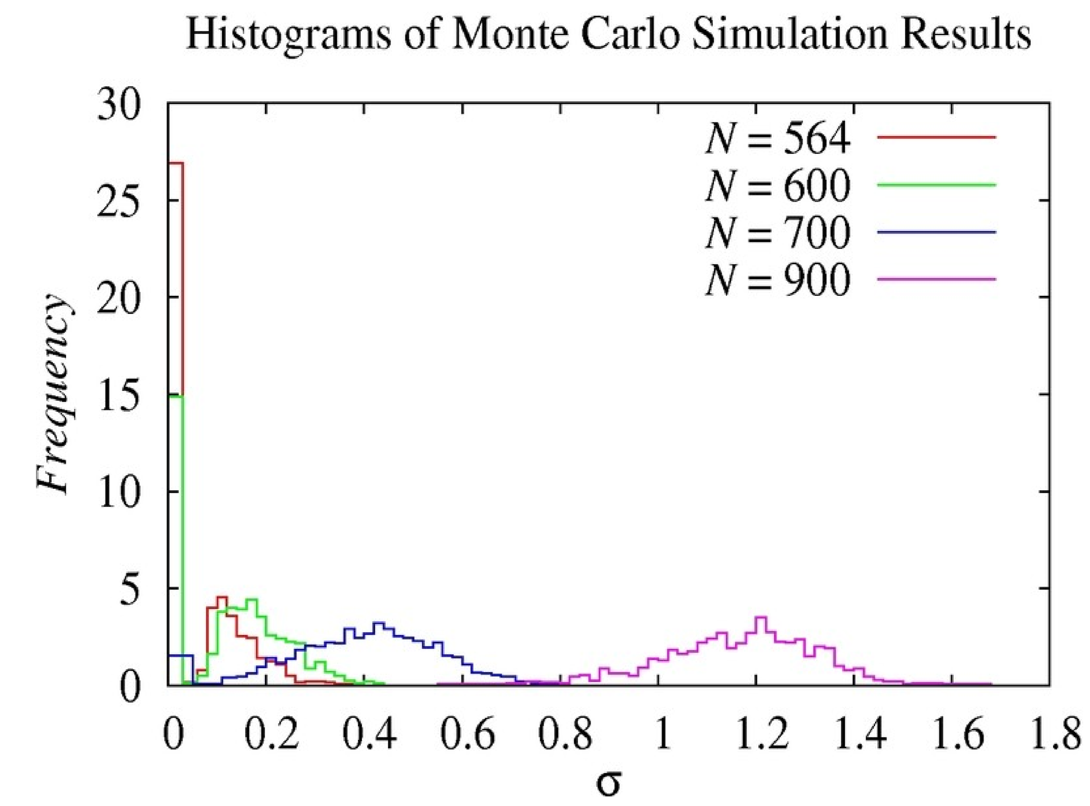
(3) Conductivity exponent in stick percolation

The average conductivity $\langle\sigma\rangle$ of a stick system varies as $\langle\sigma\rangle = (N - N_c)^t$, but only when $L \gg \xi$, where $\xi \propto |N - N_c|^{-\nu}$ is the correlation length, and $\nu = 4/3$ is universal exponent for two dimensional systems [2]. As N approaches N_c in a finite-size system, there are always finite-sample errors, even for very large systems due to the divergence of ξ. When N is well above N_c, which mean that condition $L \gg \xi$ is fulfilled, exponential low is applicable, as it shown on the figure. We obtain the critical conductivity exponent $t = 1.186 \pm 0.003$. This value is in excellent agreement with value obtained in [3].





(4) Inhomogeneous conductivity



Histograms created from Monte Carlo using 1000 simulations. Numbers of sticks is varied, the values for number of sticks are choosen close to (N = 564) and above (N = 600, 700, 900) percolation threshold.

One can observe finite size effects close to percolation point. At percolation threshold N = 564 roughly 50% realised system were not conductive. With increasing number of sticks this number decreases.

References:

[1] J. Li and S.-L. Zhang, Phys. Rev. B 79, 155434 (2009)

[2] D. Stauffer and A. Aharony, Introduction to Percolation Theory, 2nd revised ed. Taylor and Francis, London, 2003

[3] J. Li and S.-L. Zhang, Phys. Rev. B 79, 021120 (2010)

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EGEE-III
European Commission
FP7 project, 2008-2010

SCOPES
Swiss National Science
Foundation, 2009-2012

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