of three inactivating mutations (13). Although highly conserved in gene organization, as well as primary amino acid sequence of the predicted TDH open reading frame, the human TDH gene carries AG-to-GG splice acceptor mutations in exons 4 and 6, as well as a nonsense mutation within exon 6 wherein arginine codon 214 is replaced by a translational stop codon. Whereas polymorphic variation within the human population has been observed for the exon 4 splice acceptor mutation, with some individuals carrying the normal AG splice acceptor dinucleotide and others carrying the GG variant, all individuals genotyped to date carry both the splice acceptor and nonsense mutations in exon 6. Reverse transcriptase-PCR analysis of TDH transcripts expressed in human fetal liver tissue showed complete skipping of exon 4 and either complete skipping or aberrant splicing of exon 6 (fig. S8). Given that exons 4 and 6 encode segments of the enzyme critical to its function and that truncation via the nonsense codon at amino acid 214 would also be predicted to yield an inactive variant, it appears that the human gene is incapable of producing an active TDH enzyme. Remarkably, all metazoans whose genomes have been sequenced to date, including chimpanzees, appear to contain an intact TDH gene (14). Unless humans evolved adaptive capabilities sufficient to overcome three mutational lesions, it would appear they are TDH deficient.

Human ES cells grow at a far slower rate than mouse ES cells, with a doubling time of 35 hours (15). Whether the slower growth rate of human ES cells reflects the absence of a functional TDH enzyme can perhaps be tested by introducing, into human ES cells, either a repaired human TDH gene or the intact TDH gene of a closely related mammal. That strategy might work is supported by the expression in human cells of a functional form of the 2-amino-3-ketobutyrate-CoA ligase enzyme that converts the short-lived product of TDH-mediated catabolism of threonine into acetyl-CoA and glycine (Fig. 1B). It is possible that the culture conditions used to grow human ES cells do not match the ICM environment of the human embryo, in which the cell division cycle is more rapid than the 35-hour doubling time of cultured human ES cells (16). If human ES cells do not use the TDH enzyme to acquire an advantageous metabolic state for rapid growth, and if conditions can be adapted to facilitate the rapid proliferation of human ES cells in culture, the tools and approaches that we describe here may prove useful. As often happens in science, findings made in one experimental system can open avenues of investigation useful for other matters of inquiry. Finally, it is important to consider whether humans benefit from some form of selective advantage as a consequence of mutational inactivation of the TDH gene.

References and Notes

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String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

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A central problem in quantum condensed matter physics is the critical theory governing the zero-temperature quantum phase transition between strongly renormalized Fermi liquids as found in heavy fermion intermetallics and possibly in high-critical temperature superconductors. We found that the mathematics of string theory is capable of describing such fermionic quantum critical states. Using the anti–de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory. By increasing the fermion density away from the relativistic quantum critical point, a state emerges with all the features of the Fermi liquid.

Quantum many-particle physics lacks a general mathematical theory to deal with fermions at finite density. This is known as the “fermion sign problem”: There is no recourse to brute-force lattice models because the statistical path-integral methods that work for any bosonic quantum field theory do not work for finite-density Fermi systems.
The nonprobabilistic fermion problem is known to be of exponential complexity (1), and in the absence of a general mathematical framework, all that remains is phenomenological guesswork in the form of the Fermi-liquid theory describing the state of electrons in normal metals and the mean-field theories describing superconductivity and other manifestations of spontaneous symmetry breaking. This problem has become particularly manifest in quantum condensed matter physics with the discovery of electron systems undergoing quantum phase transitions that are reminiscent of the bosonic quantum critical systems (2) but are governed by fermion statistics. Empirically well-documented examples are found in the “heavy fermion” intermetallics, where the zero-temperature transition occurs between different Fermi liquids with quasi-particle masses that diverge at the quantum critical point ([6] and references therein). Such fermionic quantum critical states are believed to have a direct bearing on the problem of high-temperature (high-\(T_{c}\)) superconductivity because of the observation of quantum critical features in the normal state of optimally doped cuprate high-\(T_{c}\) superconductors ([6]; [5] and references therein).

A large part of the fermion sign problem is to understand this strongly coupled fermionic quantum critical state. The emergent scale invariance and conformal symmetry at critical points is a benefit in isolating deep questions of principle. The fundamental problem is: How does the system get rid of the scales of Fermi energy and Fermi momentum that are intrinsically rooted in the workings of Fermi-Dirac statistics (6, 7)? From another perspective, how can one construct a renormalization group with a relevant “operator” that describes the emergence of a statistics-controlled (heavy) Fermi liquid from the critical state (3), or perhaps the emergence of a high-\(T_{c}\) superconductor? Here, we show that a mathematical method developed in string theory has the capacity to answer at least some of these questions.

**String theory for condensed matter.** Our analysis makes use of the AdS/CFT correspondence: a duality relation between classical gravitational physics in a \(d + 1\)-dimensional “bulk” space-time with an anti-de Sitter (AdS) geometry and a strongly coupled conformal (quantum critical) field theory (CFT), with a large number of degrees of freedom, that occupies a flat or spherical \(d\)-dimensional “boundary” space-time. Applications of AdS/CFT to quantum critical systems have already been studied in the context of the quark-gluon plasma ([8], [9]), superconductor-insulator transitions (10–14), and cold atom systems at the Feshbach resonance ([15–17], but so far the focus has been on bosonic systems [see ([18], [19]) and references therein]. Although AdS/CFT is convenient, in principle the ground state or any response of a bosonic statistical field theory can also be computed directly by averaging on a lattice. For fermions, statistical averaging is not possible because of the sign problem. There are, however, indications that AdS/CFT should be able to capture finite-density Fermi systems as well. Ensembles described through AdS/CFT can exhibit a specific heat that scales linearly with the temperature characteristic of Fermi systems (20), zero sound (20–22), and a minimum energy for fermionic excitations (23, 24).

To address the question of whether AdS/CFT can describe finite-density Fermi systems and the Fermi liquid in particular, we compute the single charged fermion propagators and the associated spectral functions that are measured experimentally by angle-resolved photoemission spectroscopy (“AdS-to-ARPES”) and indirectly by scanning tunneling microscopy. The spectral functions contain the crucial information regarding the nature of the fermion states. These are computed on the AdS side by solving for the on-shell (classical) Dirac equation in the curved AdS space-time background with sources at the boundary. A temperature \(T\) and finite \(U(1)\) chemical potential \(\mu_{0}\) for electric charge is imposed in the field theory by studying the Dirac equation in the background of an AdS Reissner-Nordstrom black hole. We do so with the expectation that the \(U(1)\) chemical potential induces a finite density of the charged fermions. The procedure to compute the retarded CFT propagator from the dual AdS description is then well established ([8], [19]). Relative to the algorithm for computing bosonic responses, the treatment of Dirac waves in AdS is more delicate but straightforward; details are provided in (25). The equations obtained this way are solved numerically and the output is the retarded single fermion propagator \(G_{0}(\omega, k)\) at finite \(T\). Its imaginary part is the single fermion spectral function \(A(\omega, k) = -(1/\pi) \text{Im} \text{Tr}[\gamma^{0}G_{0}(\omega, k)]\) that can be directly compared with ARPES experiments (26).

The reference point for this comparison is the quantum critical point described by a zero chemical potential (\(\mu_{0} = 0\)), zero temperature (\(T = 0\)), and conformal and Lorentz invariant field theory. (Below, we use relativistic notation where \(c = 1\).) Here the fermion propagators \(\langle \Psi^{\dagger} \Psi \rangle = G(\omega, k)\) are completely fixed by symmetry to be of the form

\[
G^{\text{DFT}}_{\Delta_{\Psi}}(\omega, k) \sim \frac{1}{(\sqrt{-\omega^{2} + k^{2}})^{d-2\Delta_{\Psi}}}
\]

where \(\Delta_{\Psi}\) is the scaling dimension of the fermion field. Through the \(\text{AdS}_{d+1}/CFT_{d}\) dictionary, \(\Delta_{\Psi}\) is related to the mass parameter in the \(d + 1\)-dimensional AdS Dirac equation. Unitarity bounds this mass from below in units of the AdS radius \(mL = \Delta_{\Psi} - d/2 > -1/2\) (we set \(L = 1\) in the remainder). The choice of which value to use for \(m\) will prove essential to show the emergence of the Fermi liquid. The lower end of the unitarity-bound \(m = -1/2 + \delta\), \(\delta < 1\), corresponds to introducing a fermionic conformal operator with weight \(\Delta_{\Psi} = [(d - 1)/2] + \delta\). This equals the scaling dimension of a nearly free fermion. Even though the underlying CFT is strongly coupled, the absence of a large anomalous dimension for a fermion with mass \(m = -1/2 + \delta\) argues that such an operator fulfills a spectator role and is only weakly coupled to this CFT. We therefore use such values in our calculations. Our expectation is that the Fermi liquid, as a system with well-defined quasi-particle excitations, can be described in terms of weakly interacting long-range fields. As we increase \(m\) from \(m = -1/2 + \delta\), the interactions increase and we can expect the quasi-particle description to cease to be valid beyond \(m = 0\). For that value \(m = 0\), and beyond \(m > 0\), the naive scaling dimension \(\Delta_{\Psi} = \Delta_{\Psi}\) is marginal or irrelevant, and it is hard to see how the ultraviolet conformal theory can flow to a Fermi-liquid state, assuming that all vacuum state changes are caused by the condensation of bosonic oper-

![Fig. 1. (A) The phase diagram near a quantum critical point. Gray lines depict lines of constant \(\mu_{0}/T\); the spectral function of fermions is unchanged along each line if the momenta are appropriately rescaled. As we increase \(\mu_{0}/T\) we cross over from the quantum critical regime to the Fermi liquid. (B) The trajectories in parameter space (\(\mu_{0}/T\), \(\Delta_{\Psi}\)) studied here. We show the crossover from the quantum critical regime to the Fermi liquid by varying \(\mu_{0}/T\) while keeping \(\Delta_{\Psi}\) fixed; we cross back to the critical regime varying \(\Delta_{\Psi} = d/2\) for \(\mu_{0}/T\) fixed. The boundary region is not an exact curve but only a qualitative indication.](image-url)

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Fourier transfoments. This intuition is borne out by our results: When \( m = 0 \), the standard Fermi liquid disappears. A similar approach to describing fermionic quantum criticality (27) discusses the special case \( m = 0 \) or \( \Delta_{\text{F}} = d/2 \) in detail; an early attempt to describe the \( m = 0 \) system is (28).

The emergent Fermi liquid. With an eye toward experiment, we consider the AdS dual to a relativistic CFT in \( d = 2 + 1 \) dimensions (25). We do not know the detailed microscopic CFT, nor do we know whether a dual AdS with fermions as the sole \( U(1) \) charged field exists as a fully quantum-consistent theory for all values of \( m = \Delta_{\text{F}} - d/2 \), but the behavior of fermion spectral functions at a strongly coupled quantum critical point can be deduced nonetheless. Aside from \( \Delta_{\text{F}} \), the spectral function will depend on the dimensionless ratio \( \mu_0/T \) as well as the \( U(1) \) charge \( g \) of the fermion; we set \( g = 1 \) from here on, as we expect that only large changes away from \( g = 1 \) will change our results qualitatively. We therefore study the system as a function of \( \mu_0/T \) and \( \Delta_{\text{F}} \). Our approach is sketched in Fig. 1B. We first study the spectral behavior as a function of \( \mu_0/T \) for fixed \( \Delta_{\text{F}} < \gamma_2/2 \); then we study the spectral behavior as we vary the scaling dimension \( \Delta_{\text{F}} \) from 1 to \( \gamma_2/2 \) for fixed \( \mu_0/T \) coding for an increasingly interacting fermion. Note that our setup and numerical calculations necessitate a finite value of \( \mu_0/T \); all our results are at nonzero \( T \).

Our analysis starts near the reference point \( \mu_0/T \to 0 \), where the long-range behavior of the system is controlled by the quantum critical point (Fig. 1A). Here we expect to recover conformal invariance, as the system forgets about any well-defined scales, and the spectral function should be controlled by the branchcut at \( \omega = k \) in the Green’s function (Eq. 1): (i) For \( \omega < k \) it should vanish. (ii) At \( \omega = k \) we expect a sharp peak, which for \( \omega >> k \) scales as \( \omega^{2-\alpha_{\text{F}}} \). Figure 2A shows this expected behavior of spectral function for three different values of the momentum for a fermionic operator with weight \( \Delta_{\text{F}} = 5/4 \) computed from AdS4 following the setup in (25).

Turning on \( \mu_0/T \) while holding \( \Delta_{\text{F}} = 5/4 \) fixed shifts the center location of the two branchcuts to an effective chemical potential \( \omega = \mu_0 \); this bears out our expectation that the U(1) chemical potential induces a finite fermion density. Although the peak at the point of the negative branchcut \( \omega \sim \mu_0 - k \) stays broad, the peak at the other branchcut \( \omega \sim \mu_0 + k \) sharpens distinctly as the size of \( \mu_0/T \) is increased (Fig. 2B). We identify this peak with the quasi-particle of the Fermi liquid and its appearance as the crossover between the quantum critical regime and the Fermi-liquid regime. The spectral properties of the Fermi liquid are very well known and display a number of uniquely identifying characteristics (29, 30). If this identification is correct, all these characteristics must be present in our spectra as well.

1) The quasi-particle peak should approach a delta function at the Fermi momentum \( k = k_\text{F} \). In Fig. 2B we see the peak narrow as we increase \( k \), then peak and broaden back out as we pass \( k \sim k_\text{F} \) (recall that \( T = 0 \) is outside our numerical control and the peak always has some broadening). In addition, the spectrum should vanish identically at the Fermi energy \( \omega = E_\text{F}, k = 0 \), independent of \( k \) (Fig. 2C).

2) The quasi-particle should have linear dispersion relation near the Fermi energy with a renormalized Fermi velocity \( v_\text{F} \) different from the underlying relativistic speed \( c = 1 \). In Fig. 3 we plot the maximum of the peak \( \omega_{\text{max}} \) as a function of \( k \). At high \( k \) we recover the linear dispersion relation \( \omega = |k| \) underlying the Lorentz invariant branchcut in Eq. 1. Near the Fermi energy and Fermi momentum, however, this dispersion relation changes to a slope \( v_\text{F} = \lim_{k \to k_\text{F}} \omega/k = E_\text{F}/(k - k_\text{F}) \) clearly less than unity.

Note that the Fermi energy \( E_\text{F} \) is not located at zero frequency. Recall, however, that the AdS chemical potential \( \mu_0 \) is the bare \( U(1) \) chemical potential in the CFT. This is confirmed in Fig. 3 from the high-\( k \) behavior: Its Dirac point is \( \mu_0 \). On the other hand, the chemical potential felt by the IR fermionic degrees of freedom is renormalized to the value \( \mu_\text{F} = \mu_0 - E_\text{F} \). As is standard, the effective energy \( \phi = \omega - E_\text{F} \) of the quasi-particle is measured with respect to \( E_\text{F} \).

3) At low temperatures, Fermi-liquid theory predicts the width of the quasi-particle peak to
grow quadratically with temperature. Figure 4, A and B, shows this distinctive behavior up to a critical temperature, \( T_c/\mu_0 \approx 0.16 \). This temperature behavior directly follows from the fact that the imaginary part of the self-energy \( \Sigma(\omega, k) = \omega - k - [\text{Tr} \hat{\gamma}^j G(\omega, k)]^{-1} \) should have no linear term when expanded around \( E_F \): \( \text{Im} \Sigma(\omega, k) \sim (\omega - E_F)^2 + \ldots \). This is shown in Fig. 4, C and D. These results give us confidence that we have identified the characteristic quasi-particles at the Fermi surface of the Fermi liquid emerging from the quantum critical point.

We now discuss how this Fermi liquid evolves when we increase the bare \( \mu_0 \) (Fig. 5). Similar to the fermion chemical potential \( \mu_F \), the fundamental control parameter of the Fermi liquid, the fermion density \( \rho_F \), is not directly related to the AdS \( \mu_0 \). We can, however, infer it from the Fermi momentum \( k_F \) that is set by the quasi-particle pole via Luttinger’s theorem \( \rho_F \sim k_F^{-1} \). The more illustrative figure is therefore Fig. 5B, which shows the quasi-particle characteristics as a function of \( k_F/T \). We find that the quasi-particle velocities decrease slightly with increasing \( k_F \), rapidly leveling off to a finite constant less than the relativistic speed. Thus, the quasi-particles become increasingly heavy as their mass \( m_0 \equiv k_F/v_F \) approaches linear growth with \( k_F \). The Fermi energy \( E_F \) also shows linear growth. Suppose the heavy Fermi–quasi-particle system has the underlying canonical nonrelativistic dispersion relation \( E = k^2/(2m_F) + v_F(k - k_F) + \ldots \); in that case, the observed Fermi energy \( E_F \) should equal the renormalized Fermi energy \( E_F^{(\text{ren})} = k_F^2/(2m_F) \). Figure 5B shows that these energies \( E_F \) and \( E_F^{(\text{ren})} \)
track each other remarkably well. We therefore infer that the true zero of energy of the Fermi quasi-particle is set by the renormalized Fermi energy as deduced from the Fermi velocity and Fermi momentum.

Although the true quasi-particle behavior disappears at $T > T_c$, Fig. 5A indicates that in the limit $k_F/T \rightarrow 0$ the quasi-particle pole strength vanishes, $Z_k \rightarrow 0$, while the Fermi velocity $v_F$ remains finite; $v_F$ approaches the bare velocity $v_F = 1$. This is seemingly at odds with the heavy Fermi liquid relation $Z_k \sim m_{\text{micro}}/m_F = m_{\text{micro}}v_F/k_F$. The resolution is the restoration of Lorentz invariance at zero density. From general Fermi liquid considerations it follows that $v_F = Z_k (1 + \partial_k \text{Re } \Sigma_{f,k})$ and $Z_k = 1/(1 - \partial_m \text{Re } \Sigma_{f,k})$, where $\partial_m \text{Re } \Sigma$ refers to the momentum and energy derivatives of the real part of the fermion self-energy $\Sigma(0, k)$ at $k_F$, $E_F$, Lorentz invariance imposes $\partial_0 \Sigma = -\partial_k \Sigma$, which allows for vanishing $Z_k$ with $v_F \rightarrow 1$. Interestingly, the case has been made that such a relativistic fermionic behavior might be underlying the physics of cuprate high-$T_c$ superconductors (31).

Finally, we address the important question of what happens when we vary the conformal dimension $\Delta \varphi$ of the fermionic operator. Figure 6 shows that the Fermi momentum $k_F$ stays constant as we increase $\Delta \varphi$. This completes our identification of the new phase as the Fermi liquid: It indicates that the AdS dual obeys Luttinger’s theorem, if we can interpret the conformal dimension of the fermionic operator as a proxy for the interaction strength. We find furthermore that the quasi-particle pole strength vanishes as we approach $\Delta \varphi = \gamma_2$. This confirms our earlier assumption that it is essential to study the system for $\Delta \varphi < d/2$ and that the point $\Delta \varphi = d/2$, where the naive fermion bilinear becomes marginal, signals the onset of a new regime. Because the fermion bilinear is marginal at that point, this ought to be an interesting regime in its own right, and we refer to (27) for a discussion thereof (32). Highly remarkable is that the pole strength vanishes in an exponential fashion rather than the anticipated algebraic behavior (6, 7). This could indicate that an essential singularity governs the critical point at $\Delta \varphi = d/2$, and we note that such a type of behavior was identified by Lawler et al. in their analysis of the Pomeranchuk instability in $d = 2 + 1$ dimensions using the Haldane patching bosonization procedure (33). Note that this finite $\mu_0/T$ transition as we vary $\Delta \varphi$ has no clear symmetry change, similar to (7). However, this may be an artifact of the fact that our theory is not quantum mechanically complete (25). Note also that the quasi-particle velocity and the renormalized Fermi energy $E_F = Z_k(k-k_F)$ stay finite at the $\Delta \varphi = \gamma_2$ transition with $Z \rightarrow 0$, which could indicate an emergent Lorentz invariance for the reasons discussed above.

Concluding remarks. We have presented evidence that the AdS dual description of strongly coupled field theories can describe the emergence of the Fermi liquid from a quantum critical state, as a function of both density and interaction strength, as encoded in the conformal dimension of the fermionic operators. From the AdS gravity perspective, it was unclear whether this would happen. Sharp peaks in the CFT spectral function correspond to so-called quasi-normal modes of black holes (34), but Dirac quasi-normal modes have received little study [see, e.g., (35)]. It is remarkable that the AdS calculation processes the Fermi-Dirac statistics essential to the Fermi liquid correctly. This is manifested by the emergent renormalized Fermi energy and the validity of Luttinger’s theorem. The AdS gravity computation, however, is completely classical without explicit quantum statistics, although we do probe the system with a fermion. It would therefore be interesting to fully understand the AdS description of what is happening, in particular how the emergent scales

Fig. 5. The quasi-particle characteristics as a function of $\mu_0/T$ for $\Delta \varphi = 5/4$. (A) The change of $k_F$, $v_F$, $m_F$, $E_F$, and the pole strength $Z_k$ (the total weight between half-maxima) as we change $\mu_0/T$. Beyond a critical value ($\mu_0/T$), we lose the characteristic $k^2$ broadening of the peak and there is no longer a real quasi-particle, although the peak is still present. For the Fermi liquid, $k_F/T$ rather than $\mu_0/T$ is the defining parameter. (B) We can invert this relation, and (B) shows the quasi-particle characteristics as a function of $k_F/T$. Note the linear relationships of $m_F$ and $E_F$ to $k_F$ and that the renormalized Fermi energy $E^{(\text{ren})} = k_F^2/(2m_F)$ matches the empirical value $E_F$ remarkably well.

Fig. 6. The quasi-particle characteristics as a function of the Dirac fermion mass $-\gamma_2 < m < 0$ corresponding to $1 < \Delta \varphi < \gamma_2$ for $\mu_0/T = -30.9$. The upper panel shows the independence of $k_F$ of the mass. This indicates Luttinger’s theorem if the anomalous dimension $\Delta \varphi$ is taken as an indicator of the interaction strength. Note that $v_F$ and $E_F$ both approach finite values as $\Delta \varphi \rightarrow \gamma_2$. The lower panel shows the exponential vanishing pole strength $Z$ (the integral between the half-maxima) as $m \rightarrow 0$. 

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Radio Imaging of the Very-High-Energy γ-Ray Emission Region in the Central Engine of a Radio Galaxy

The VERITAS Collaboration, the VLBA 43 GHz M87 Monitoring Team, the H.E.S.S. Collaboration, the MAGIC Collaboration

The accretion of matter onto a massive black hole is believed to feed the relativistic plasma jets. The substructures of the jet, which are expected to scale with the Schwarzschild radius $R_s$ of the black hole ($7$), are resolved in the x-ray, optical, and radio wave bands (1). High-frequency radio very-long-baseline interferometry (VLBI) observations with resolution under a milli–arc second (milli–arc sec) are starting to probe the collimation region of the jet (9). With its proximity, bright and well-resolved jet, and very massive black hole, M87 provides a unique laboratory in which to study relativistic jet physics in connection with the mechanisms of VHE γ-ray emission in AGN. VLBI observations of the M87 inner jet show a well-resolved, edge-brightened structure extending to within 0.5 milli–arc sec (0.04 pc or 70 $R_s$) of the core. Closer to the core, the jet has a wide opening angle, suggesting that this is the collimation region (9). Generally, the core can be offset from the actual location of the black hole by an unknown amount (10), in which case it could mark the location of a shock structure or the region where the jet becomes optically thin. However, in the case of M87, a weak structure

$E_F^g$ and $k_F^g$ feature in the geometry. An early indication of such scales was seen in (24, 30) in a variant of the story that geometry is not universal in string theory: The geometry depends on the probe used, and different probes experience different geometric backgrounds. The absence of these scales in the general relativistic description of the AdS black hole could thus be an artifact of the Riemannian metric description of space-time.

Regardless of these questions, AdS/CFT has shown itself to be a powerful tool to describe finite-density Fermi systems. The description of the emergent Fermi liquid presented here argues that AdS/CFT is uniquely suited as a computational device for field theory problems suffering from fermion sign problems. AdS/CFT represents a rich mathematical environment and a new approach to qualitatively and quantitatively investigate important questions in quantum many-body theory at finite fermion density.

References and Notes
25. See supporting material on Science Online.
26. ARPS/Fermi-surface measurements assume that electrons are the only relevant charged objects. If this is so, then it measures the electron (i.e., fermion) spectral function. This spectral function is what we compute here, even though in our AdS setup the fermions are almost certainly not the only charged objects.
32. Note that the $m = 0$ spectral peak discussed in (27) is therefore not the peak we identified with the quasi-particle state. See (25).
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